2010 World Mathematics Team Championship

Middle School Level

Team Round

1. $\sqrt{2011\sqrt{2010\sqrt{2009\times 2007+1}+1}+1}=$ ________.

2. If a and b are rational numbers and $x=\dfrac{\sqrt{5}+1}{2}$ is a solution for the equation $x^3-ax-b=0$, then $a=$ ________ and $b=$ ________.

3. Given that real numbers a, b and c satisfy $a-b=\dfrac{7}{4}ab$ and $b-c=\dfrac{7}{4}bc$. When a takes on values of, in order, $1, \dfrac{2009}{2010}, \dfrac{2008}{2010}, \cdots, \dfrac{3}{2010}, \dfrac{2}{2010}, \dfrac{1}{2010}, 0, -\dfrac{1}{2010}, -\dfrac{2}{2010}, -\dfrac{3}{2010}, \cdots, -\dfrac{2008}{2010}, -\dfrac{2009}{2010}, -1$, b and c would take on a total of ________ negative numbers as values.

4. Given that x, y, z are integers. If $x>y>z$ and $2^x+2^y+2^z=4.625$, then $xyz=$ ________.

5. As shown in figure 1, $\odot O$ and the hypotenuse of Rt$\triangle AOB$ intersect at C and D. If C and D trisect AB and the radius of $\odot O$ is 5, then $AB=$ ________.

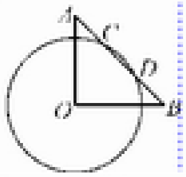


figure 1

6. A store is purchasing a product to sell. This product costs the store \$10 each. If the store sells this product for \$15 each, then it can sell 120 of this product. In general, if the sale price goes up by $a\%$, then the sales will also go down by $a\%$. To get the largest profit, the store should set the sale price at \$ ________ each.

7. Suppose that the proportion of copper to iron is 1 : 2 in alloy A of 2 kg, and 3 : 2 in alloy B of 7 kg. If we are going to melt some alloy A and alloy B together and form a new alloy so that this new alloy has a proportion of copper to iron as 6 : 5, then the maximal amount of this new alloy we can form is ________ kg.

8. Given a and b are 1-digit numbers, and $\overline{45ab}$ is a 4-digit number. To make the absolute difference between the two numbers $27(a+b)^2$ and $\overline{45ab}$ as small as possible, use $a=$ ________ and $b=$ ________.

9. Suppose $f(x)$ is a polynomial of x. If $f(x)$ has a remainder of 3 when it is divided by $2(x-1)$ and $2f(x)$ has a remainder of -4 when it is divided by $3(x+2)$, then $3f(x)$ has a remainder of ________ when it is divided by $4(x^2+x-2)$.

10. As it is shown in figure 2, point P is a point inside of the regular hexagon ABCDEF. Use this point to divide this hexagon into 6 triangles $\triangle PAB$, $\triangle PBC$, $\triangle PCD$, $\triangle PDE$, $\triangle PEF$, $\triangle PFA$ with six areas S_1, S_2, S_3, S_4, S_5, S_6, respectively. If $S_1-S_2+S_3=1$, then $S_5+S_6=$ ________.

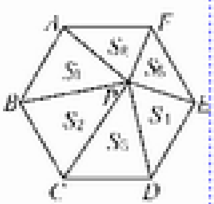


figure 2

11. Let P be the product of 3 consecutive odd positive integers. Then the largest integer that can divide into all such P is ________.

12. Suppose a, b, c are the lengths of a triangle's three sides. Also, a is an integer and it is also the largest of the three lengths. If a satisfies the set of equations

$$a^2 + 2b - 10c + 10 = 0 \quad ①$$

$$a + 2b - 5c + 7 = 0 \quad ②$$

then $a =$ ________, $b =$ ________, $c =$ ________.

13. As it is shown in figure 3, if E and F are points on the square ABCD's sides BC and CD, respectively, and $\angle EAF = 45°$, then the smallest possible value for $\frac{EF}{AB}$ is ________.

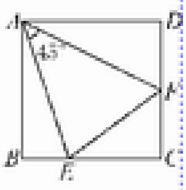


figure 3

14. Suppose the straight line $y = (1-k)x + k (k < 1)$ intersects the hyperbola $y = \frac{6}{x}$ at $A(x_1, y_1)$ in the 1^{st} Quadrant and at $B(x_2, y_2)$ in the 3^{rd} Quadrant. If we draw perpendicular lines from A and B to the x axis and they intersect at points M and N, respectively, then the area of the quadrilateral AMBN has the smallest value of ________ when $k =$ ________.

15. Consider the 5×5 square grid in figure 4. ________ squares can be formed by using some of these 36 grid points as vertices.

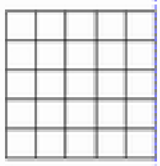


figure 4

16. The roots of the set of equations

$$\begin{cases} x_1 + x_2 = x_2 + x_3 = \cdots = x_{2010} + x_{2011} = 1 \\ x_1 + x_2 + x_3 + \cdots + x_{2010} + x_{2011} = 2011 \end{cases}$$

are ________.

17. If $a + b + c = 0$, then $\left(\frac{b-c}{a} + \frac{c-a}{b} + \frac{a-b}{c}\right)\left(\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b}\right) =$ ________.

18. As in figure 5, isosceles right triangle ABC has its hypotenuse AC on the straight line MN. Its sides BA and BC both have length of 3. If we rotate this triangle, without sliding, to the right along the line for one revolution until AC is on MN again, then point A travelles a distance of ________ during this rotation.

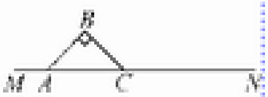


figure 5

19. Consider the circle ⊙O in figure 6. The diameter of ⊙O is AB = 20. Point P is outside of ⊙O and PC and PB are tangents to ⊙O at C and B, respectively. Chords CD ⊥ AB at E. PA intersects CD at M. If $\frac{AE}{EB} = \frac{1}{4}$, then the area of $\triangle PCM$ is ________.

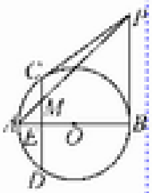


figure 6

20. Suppose there are 6 cities (A, B, C, D, E, F) and 6 islands (a, b, c, d, e, f) with ferries connecting them. Each city must have at least one ferry connection to one island. If the total number of connections from each city A, B, C, D, E are 5, 4, 3, 2, 2, respectively, and if the total number of connections from each island a, b, c, d, e are 4, 3, 2, 1, 1, respectively, then the total number of connection lines to cities from island f is ________.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 1 • A

It took 24 hours for a piece of wooden log floating downstream on a river from Location A to Location B. A boat has a speed of 18 km/hour when the water speed is 0. This boat can go from A to B in a time that is $\frac{3}{5}$ of the time it took the boat to go from B to A. Then the distance between Location A and Location B is ________ km.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 1 • B

Let $A=$ the answer passed from your teammate. Rearrange the digits from the number $2A$ which is a 3-digit number and form new numbers. If S is the sum of numbers from all the possible rearrangements and we also know that $S=\left[\frac{n(n+1)}{2}\right]^2-3(a+b+c)$ where a, b, c are the digits of $2A$ and n is a natural number, then $n=$ ________.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 2 • A

If $x < 0$ and $y < 0$ and x [illegible] $y =$ [illegible] $\sqrt{xy}$, then $\frac{x}{y} =$ __________.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 2 • B

Let $b =$ the answer passed from your teammate. Suppose $x^2 - y^2 + ax - by -$ [illegible] can be factored into linear factors of x and y. Then $a =$ __________.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 3 • A

If three line segments of lengths $\sqrt{n}$, $1+\sqrt{n+2}$ and $\sqrt{n}+\sqrt{n(n+2)}$ are used to form a triangle, then the possible natural numbers $n=$ __________.

2010 World Mathematics Team Championship

Middle School Level

Relays Round

Round 3 • B

Let $m=$ the larger number of the answer passed from your teammate. Find the three interior angles of the triangle that has $\sqrt{m}$, $1+\sqrt{m+1}$, and $\sqrt{m}+\sqrt{m(m+1)}$ as the lengths of its three sides.

2010 World Mathematics Team Championship

Middle School Level

Individual Round

Round 1

1. The sum of all the coefficients of the expanded expression $(xy+2x^2y^2+3x^3y^3+4x^4y^4+5x^5y^5)^2$ is ________.

2. There are ________ pairs of integers (m, n) that satisfy $m^2-n^2=625$.

3. As in figure 1, $AB=AC$ and $DEFG$ is a rectangle. Then $\angle DHC=$ ________.

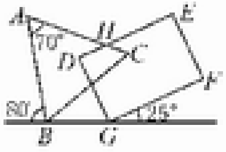


figure 1

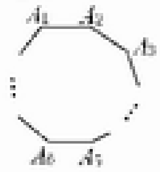


figure 2

4. As in figure 2, an equiangular polygon with an even number of sides. Let A_1A_2 be the first side, A_2A_3 be the second side, and so on. If the fifth side is parallel to the first side, then the degree of one of its interior angles is ________.

2010 World Mathematics Team Championship

Middle School Level

Individual Round

Round 2

1. There are ________ possible acute triangles with perimeter not more than 2010 and the lengths of its sides are consecutive natural numbers.

2. If $f(x)=|x^2-9\cdot 10^{499}x-10^{999}|$, then $f(11)$ has ________ different factors among the numbers 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12.

3. Given points $A(1,1)$, $B(-1,2)$ and point C is on the straight line $y=-x$, then the shortest perimeter for triangle ABC is ________.

4. Given a and b are 1-digit numbers, and $\overline{58ab}$ is a 4-digit number. To make the absolute difference between $27a^3$ and $\overline{58ab}$ as small as possible, $a=$ ________, $b=$ ________.

2010 World Mathematics Team Championship

Middle School Level

Individual Round

Round 3

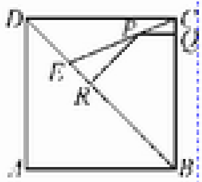
1. Given $\sqrt{x}=\frac{1-a}{2}$ (a is a constant). Simplify $\sqrt{x+a}-\sqrt{x-a+2}=$ __________.

2. The whole number 12345⋯9899100 has a remainder of __________ when it is divided by 9.

3. For $n=1,2,3\cdots$, how many of the following choices represent a number that cannot be divided by 6 evenly?

① n^3-n. ② $8n^3-2n$. ③ $2n^3+3n^2+n$. ④ n^5-n.

4. As shown in the figure, E is on the diagonal BD of a 1×1 square $ABCD$ and $BE=BC$. P is any point on CE and $PQ\perp BC$ at ponit Q, $PR\perp BE$ at point R. Then $PQ+PR$ has a total length of __________.



2010 World Mathematics Team Championship

Middle School Level

Individual Round

Round 4

1. Suppose each of $x_1, x_2, \cdots, x_n$ can take on one of three numbers -1, 2 and -3. If $x_1+x_2+\cdots+x_n=3$ and $x_1^2+x_2^2+\cdots+x_n^2=15$, then $x_1^5+x_2^5+\cdots+x_n^5=$ __________.

2. A point in an x-y coordinate system is called a *lattice point* with integer coordinates. If the vertices of a convex n-sided polygon are all lattice points and its interior or sides do not contain any lattice points, then $n=$ __________.

2010 **World Mathematics Team Championship**

Middle School Level

Individual Round

Round 5

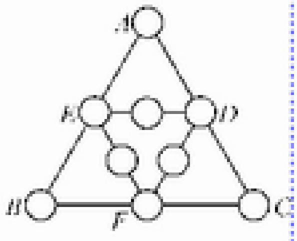


figure 1

1. Insert numbers 1,2,3,…,9 into the 9 circles of figure 1, respectively, so that the sum of the numbers in the three circles in each side of triangle ABC and triangle DEF are all equal to 18. There are a total of __________ ways to fill those 9 numbers.

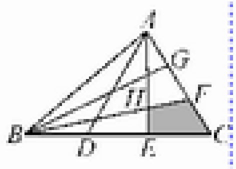


figure 2

2. As in figure 2, the area of triangle ABC is 1. D and E are trisection points for side BC. F and G are trisection points for side AC. AE and BF intersect at H. Then the area of the quadrilateral $CEHF$ is __________.