Number Theory I:
The Last Digit
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Problem 1.   What is the last digit of 7^{2012}?

Problem 2.   Establish a pattern so that you can quickly determine the last digit of any integer to a positive integer exponent.

Problem 3.   How could you categorize the different kinds of patterns that emerge? What conjectures can you make?

Problem 4.   What is the last digit of -19?

Problem 5.   If we reinterpret "last digit" to mean "remainder when divided by 10", we can quickly explore last digits in cases where we don't happen to have ten fingers. What patterns do we discover? Let's first try 8 fingers, then 7, then 11, and then see what other numbers we think might be interesting.

Problem 6.   Is there symmetry?

Problem 7.   What categories can we put these different patterns into? How important is it to know how many fingers we happen to have?

Problem 8.   How long does it take to determine the remainder when you divide 2^{2013} by 13? Once you've done that, can you quickly find the answer for 6^{2013}? If we had done these in the opposite order, would it have mattered?

expt	1	2	3	4	5	6	7	8	9	0
1st										
2nd										
3rd										
4th										
5th										

expt	1	2	3	4	5	6	7	0
1st								
2nd								
3rd								
4th								
5th								

expt	1	2	3	4	5	6	0
1st							
2nd							
3rd							
4th							
5th							
6th							
7th							

expt	1	2	3	4	5	6	7	8	9	10	0
1st											
2nd											
3rd											
4th											
5th											
6th											
7th											
8th											
9th											
10th											