PRIMES AND PROOFS, 3/12/2013

1) Use the Sieve of Eratosthenes to find all prime numbers less than 100:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2) Use the Sieve of Eratosthenes to find all prime numbers less than 225:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130	131	132	133	134	135
136	137	138	139	140	141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160	161	162	163	164	165
166	167	168	169	170	171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190	191	192	193	194	195
196	197	198	199	200	201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220	221	222	223	224	225

3) Let's practice a proof by contradiction. You will prove that $\sqrt{2}$ is not a rational number. I'll guide you along. Fill in the details.

a) Suppose that $\sqrt{2}$ **is** a rational number. That means that there are two integers x and y which share no factors other than 1 such that.....

b) Square both sides of the expression you came up with in (a) and multiply both sides by y^2. What do you get?

c) Now go through the following four possibilities and find a contradiction of some sort for each one: both x and y are even; x is odd and y is even; x is even and y is odd; both x and y are odd.

4) Can you find a prime factorization for the following numbers? Are there other possible prime factorizations?

a) 2940
b) 4568
c) 571
d) 4572
e) 3690

5) Let's practice a proof by induction. We will show that $1 + 2 + \cdots + n = \frac{n^2+n}{2}$. Note that there is more than one way to prove this, but we'll try the induction way.

a) Check the base case of $n = 1$.

b) What should the "inductive hypothesis" be?

c) What do we want to show given the inductive hypothesis?

d) Show it!