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Investigating some sequences...

1. 1, 3, 5, 7, ...
2. 1, 4, 7, 10, ...
3. 1, 2, 4, 7, 11, ...
4. 1, 3, 6, 10, ...
5. 1, 4, 9, 16, ...
6. 1, 1, 2, 3, 5, ...

A. The Handshake Problem

At the first math circle meeting of a new year, in order to get to know each other, every student wants to *virtually* shake hands with every other student in the room. How many total handshakes will there be?

B. The city of Townsville is 6 blocks by 6 blocks, arranged in a diamond. You live at the top-most corner. When travelling to visit your neighbors, Townsville traffic laws dictate that you may only move downward, either to the right or to the left. How many possible paths can you take to visit your cousin in the leftmost corner, your grandmother in the rightmost corner, and the grocery store in the bottom corner?

C. What are the odds of getting any particular outcome when you toss one coin? What about two? Three? Four? Five?

D. If we realign Pascal's Triangle to occupy the lower left half of a square grid, can we see anything new?

E. Find the first twenty terms of the sequence 1, 1, 2, 3, 5, ... Find the ratio of the last term to the next-to-last term, that is $n/(n-1)$.
Now choose any two numbers between 1 and 20. Following the rule from the first sequence, and starting with these two numbers, find the first twenty terms of the sequence. Find the ratio of the last term to the next-to-last term.

F. Making Magic with Fibonacci-like Sequences

Have a friend pick two numbers between 1 and 20 as in the previous problem. Following the rule from the Fibonacci sequence, have them find the first 10 terms. Don't look! Then, when they're done, look at their numbers and immediately write down the sum of all 10 numbers.

E. Which Fibonacci numbers are even? What about multiples of three? Five? Eight? Can we express our findings as a general rule?