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1. A falling number is an integer whose decimal representation has the property that each digit except the units digit is larger than the one to its right. For example 96521 is a falling number but 89642 is not. How many n -digit falling numbers are there, for $n = 1, 2, 3, 4, 5, 6$, and 7 ?

2. A sandwich shop makes sandwiches with one meat, one cheese, one sauce, and two toppings. Their meats are turkey, roast beef, ham, and tofu. They have cheddar and jack cheeses. They have mayonnaise, mustard, and ranch. They have lettuce, tomatoes, pickles, and olives for toppings. They make sandwiches on white or wheat bread and you can order a half sandwich or a whole sandwich. How many different sandwiches can you order?

3. You are ordering dinner at your favorite restaurant. You want a drink, an appetizer, a main course, 2 different side items, and a dessert. If there are 10 choices for drinks, 5 appetizers, 6 main courses, 8 side items, and 5 desserts, in how many ways can you order your meal?

4. If 30 students have entered the science fair, how many possible ways could 1st, 2nd, and 3rd place be awarded?

5. A math club consists of 20 students. They want to choose a club president, vice president, secretary, and treasurer. How many different club governments can they elect?

6. The Handshake Problem: Revisited

At the first math circle meeting of a new year, in order to get to know each other, every student wants to virtually shake hands with every other student in the room. How many total handshakes will there be?

7. How many numbers can be expressed as a sum of four distinct members of the set $\{17, 21, 25, 29, 33, 37, 41\}$?

8. How many numbers can be obtained as the product of two or more of the numbers 3, 4, 4, 5, 5, 6, 7, 7, 7?

9. How many integers can be obtained as a sum of two or more of the numbers 1, 3, 5, 10, 20, 50, 82?

10. How many even 3-digit numbers have no repeating digits?

11. How many different ways can we permute the letters in the word GUITAR?

12. What about FANTASTIC?

13. How about MISSISSIPPI?

14. If we are trying to choose 4 students out of a class of 20 for a MATHCOUNTS competition team, how many different teams can we make?

15. There are 12 boys and 14 girls in a speech class. In how many different ways can 3 boys and 3 girls be chosen for a debate team?

16. How many 10-bit strings (of 0's and 1's) have exactly 3 0's?