Number Theory III:
The Last Digit

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We have observed many patterns in the tables on the next few pages. Today we will explain some of them. You may wish to make a few more tables like these for other "moduli", like 5, 6, or 13, in addition to the ones we have already created. Here's a few observations and questions; I hope we will notice and wonder about some more!

1. $n^{\text{even}} = (-n)^{\text{even}}$, $(-n)^{\text{odd}} = -n^{\text{odd}}$

2. For what numbers n does the "Dividing by n" table have a row that's the same as the first powers? When such a row exists, how far down the chart will it be?

3. For what numbers n does the "Dividing by n" table have a column that contains all the possible remainders (except 0)?

4. What do you notice if you only include the columns that contain at least one 1 somewhere in them?

5. What happens when you take some of the cycles and arrange them in a circle? What does this tell you about the patterns formed by the other numbers in each cycle?

6. How can you tell when there will be a value such that $n^2 = n$? $n^3 = n$? Some other power of n?

7. How about $n^2 = 1$? Some other power of n equal to 1? The smallest exponent that gives 1?

Dividing by 10										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**0**
1st	1	2	3	4	5	6	7	8	9	0
2nd	1	4	9	6	5	6	9	4	1	0
3rd	1	8	7	4	5	6	3	2	9	0
4th	1	6	1	6	5	6	1	6	1	0
5th	1	2	3	4	5	6	7	8	9	0

Dividing by 8								
1st	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**0**
2nd	1	4	1	0	1	4	1	0
3rd	1	0	3	0	5	0	7	0
4th	1	0	1	0	1	0	1	0
5th	1	0	3	0	5	0	7	0

Dividing by 7							
1st	**1**	**2**	**3**	**4**	**5**	**6**	**0**
2nd	1	4	2	2	4	1	0
3rd	1	1	6	1	6	6	0
4th	1	2	4	4	2	1	0
5th	1	4	5	2	3	6	0
6th	1	1	1	1	1	1	0
7th	1	2	3	4	5	6	0

Dividing by 11											
1st	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**0**
2nd	1	4	9	5	3	3	5	9	4	1	0
3rd	1	8	5	9	4	7	2	6	3	10	0
4th	1	5	4	3	9	9	3	4	5	1	0
5th	1	10	1	1	1	10	10	10	1	10	0
6th	1	9	3	4	5	5	4	3	9	1	0
7th	1	7	9	5	3	8	6	2	4	10	0
8th	1	3	5	9	4	4	9	5	3	1	0
9th	1	6	4	3	9	2	8	7	5	10	0
10th	1	1	1	1	1	1	1	1	1	1	0