PRIMES AND PROOFS, 4/9/2013

1) From last time: Suppose we are given three positive whole numbers (integers) a, b, c. Suppose that $a \times b$ is divisible by c but b and c share no factors bigger than 1. True or false: Then a must be divisible by c.

2) We will now prove a "corollary" of the theorem in the last problem. Again, suppose that a and b are positive whole numbers. We will show that if p is a prime number and $a \times b$ is divisible by p then a is divisible by p or b is divisible by p (or both).

a) If we have that both p and a and p and b share some common factors bigger than 1 then we are done. Why?

b) Suppose that p shares no common factor with a. How do we know that then b must be divisible by p? Hint: think about what we proved in the last problem.

c) Suppose that p shares no common factor with b. How do we know that then a must be divisible by p? Does this finish off the proof?

Optional Problem: We say that a positive fraction is written in *reduced form* if it is written as $\frac{a}{b}$ where a and b are positive whole numbers that share no factor bigger than 1. Is it true that there is only one way to write any given positive fraction in reduced form? Prove it. You might need the theorem you proved in problem 3.

3) Write out a list of prime numbers less than or equal to 45 which can be written as a sum of two squares of whole numbers. Try to make sure your list is complete.

4) Write out a list of whole numbers less than or equal to 45 which can be written as a sum of two squares of whole numbers. Try to make sure your list is complete.

5) Write out a list of whole numbers less than or equal to 45 which can be written as a sum of three squares of whole numbers. Try to make sure your list is complete.

6) Write out a list of whole numbers less than or equal to 45 which can be written as a sum of four squares of whole numbers. Try to make sure your list is complete.