Mathematical Magic for Muggles   April 16, 2013

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Here are several easy-to-perform feats that suggest supernatural powers such as telepathy, "seeing fingers," predicting the future, photographic memory, etc. Each trick uses simple mathematical ideas that allow information to flow effortlessly and sneakily, among them

- simple, efficient "coding"

- parity and other invariants

- symmetry

- probability

One can approach these activities in many ways. At first, you may want to figure out HOW to do a trick. Then, you want to know WHY it works. Finally, you should strive to understand REALLY WHY it works: is there a simple theme or principle behind your possibly complex explanation? Look for simple and general guiding principles. If you REALLY understand a trick, you should be able to create new tricks of your own!


Mostly about Coding and Communication

1 *Warm-up: Fingers That Can See.* The Magician deals cards on a table (not in a pile), placing them face up or face down on the command of the Participant, and stops dealing when the Participant says so.

Then the Magician is blindfolded. The Magician proceeds to put the cards into two piles, using his magical seeing fingers, so that, miraculously, each pile has exactly the same number of face-up cards!

2 *Warm-up: Buried in the Sand.* (Told to me by an 11-year-old girl.) Four unlucky people, labeled 1, 2, 3, 4, from left to right, are buried in sand up to their chins, as illustrated below.
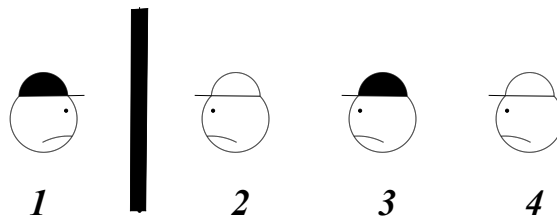
There is a wall be separating #1 from #2; they are both facing this wall and that is all that they can see. However, #4 can see the back of the head of #3 and #2, and #3 can see the back of the head of #2. The evil person who imprisoned them put a hat on each person in such a way that no one can see the color of the hat that they are wearing, but can see the hats of people in front of them (unless there is a wall in the way). The evil person then said the following:

> I have put hats on each of you. Two hats are white, and two are black. If any of you can determine the color of your hat, and then explain your reasoning to me, I will free you all. If you say the wrong color, or if you merely guess correctly, and your reasoning isn't logical, then all of you will die.

Assume that all four people are intelligent and equally so. What happens? Explain.

3 *Zvonkin's Magic Table.* This trick is adapted from A. Zvonkin's book *Math From 3 to 7*, which I helped to translate and edit. Zvonkin ran a math circle for small kids in Moscow and entertained them by having them cover any four consecutive numbers in the table below (vertical or horizontal), and then he would instantly determine the sum! Was it a feat of memory? Telepathy?

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5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
7 1 7 2 3 7 1 7 2 3 7 1
2 7 6 1 4 2 7 6 1 4 2 7
5 6 1 6 2 5 6 1 6 2 5 6
5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
7 1 7 2 3 7 1 7 2 3 7 1
2 7 6 1 4 2 7 6 1 4 2 7
5 6 1 6 2 5 6 1 6 2 5 6
5 6 1 6 2 5 6 1 6 2 5 6
1 0 5 5 9 1 0 5 5 9 1 0
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4 *Concentration.* The Participant deals out cards in a 4×4 grid, putting some face up, some face down randomly. Then the Magician deals a few more cards, adding one more row and one more column. The Magician is then blindfolded and the Participant picks one card in the grid and turns it over (i.e., if it was face up, now it is face down, and if it was face down, now it is face up). The Magician takes off the blindfold and is miraculously able to spot the altered card. How?

5 *More Hats.* Ten people are now the victims of the Evil Villain from Problem #2. This is what E. V. tells them:

> In a few minutes, you will line up, and I will put hats on you. The hats will be black or white. You will be able to see the hat colors of the people in front of you, but you won't be able to see your hat or the hats of the people behind you. Starting at the back of the line, I will ask you to say your hat color. You are

allowed to say the single word "black" or "white" just once, and otherwise, you are not allowed to communicate with each other at all.

If you answer incorrectly, I will kill you instantly and loudly, so that the people in front of you will know. If you answer correctly, you will be spared.

You can take a few minutes to confer on a strategy before I line you up. After that, you may not communicate with each other in any way except when you state your hat color.

What is the best strategy? How many people can be saved?

6 *Telepathic Teacher*. The Teacher, blindfolded, asks a Student to write a largish (four to six digits, say) number on the board. The student is instructed to then write the number backwards, and to subtract the smaller of the two from the larger, with other students quietly checking the work to make sure it is perfect.

Then the Teacher asks the student to circle one digit in the answer, and then say what the other digits are. The class is then asked to concentrate deeply on the circled digit. The Teacher is able, with high probability, to correctly name the digit.

How is this done? Why is it only with "high probability?"

7 *The Kruskal Count*. This telepathy trick can be done with cards or numbers. With cards, the Magician deals out an entire deck face up on a table, and asks the participant to mentally pick one of the first dozen or so cards and then use that card to tell him or her where to go next. If the card is an Ace, move one spot to the next card. If it's 2 through 9, go that many places. If it's a face card, move the number of letter of the card (i.e., Jack or King means move four, Queen means move five). Keep doing this until you can go no further. For example, if you start with the Jack of Hearts, you then move 4 cards down and perhaps that is an Ace of clubs. Then you move to the next card, the 7 of spades, and move 7 down, etc.

When the participant gets to the final card (the one where you cannot go further, because you'd go past the last card in the deck), he or she thinks hard about it. And the Magician manages to deduce the card.

The trick can also use a random list of numbers, or a semi-random one, such as the digits of π below.

3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3	2	3	8	4
6	2	6	4	3	3	8	3	2	7	9	5	0	2	8	8	4	1	9	7
1	6	9	3	9	9	3	7	5	1	0	5	8	2	0	9	7	4	9	4
4	5	9	2	3	0	7	8	1	6	4	0	6	2	8	6	2	0	8	9
9	8	6	2	8	0	3	4	8	2	5	3	4	2	1	1	7	0	6	7

With a number table, the rule is simpler: Pick any starting point in the row, and move that many places, unless you hit 0, in which case you move one place. For example, if you start with the second digit (1), you move one place, to 4, then 4 more places, to 2, then 2 places, to 5, etc. Once again, the Participant mentally chooses a starting point, concentrates on the ending number, and the Magician magically guesses it!

Mathematical Card Tricks

Several of these tricks were researched, perfected, and classroom-tested this winter at the San Francisco Math Circle by SFSU grad students Jessica Delgado and Kelly Walker. I am indebted to them. In turn, they (and I) are also indebted to the recent *Magical Mathematics*, by Persi Diaconis and Ron Graham (Princeton University Press, 2012).

8 *Hummer Shuffle Tricks*. The three tricks below all employ the "Hummer Shuffle," which consists of picking up the first two cards of a deck, turning the two cards over, and replacing them on the top of the deck (i.e., card #1 becomes card #2 and card #2 becomes card #1, and both get turned over), followed by cutting the deck (you take the top n cards, where n is up to you, and lift them off the deck, then place them at the bottom, without turning the n cards over, so that now the top card is the previous $(n+1)$st and the bottom card is the previous nth card, etc. After doing a bunch of Hummer Shuffles, the cards in a deck are hopelessly messed up, since not only is the order permuted, but some of the cards will be face up and some will be face down. However, this shuffle is surprisingly orderly, as you will see.

   (a) *Baby Hummer*. This trick only uses four cards. The Participant takes four cards, all facing the same way, and sneaks a peek at the bottom card. Then the Participant does the following:

      1. Take the top card and place it on the bottom
      2. Turn the current top card face up
      3. Perform several Hummer Shuffles
      4. Turn over the top card and put it on bottom
      5. Put the current top card on the bottom without turning it over
      6. Turn the top card over and leave it on top

   Now spread the cards out and three cards will be facing one way and your original bottom card with be facing the other!

   (b) *Nearly Perfect Mind Reading?* The Magician gives the Participant ten cards from A to 10, in order. The Participant then performs several Hummer Shuffles, thoroughly messing up the cards. The Magician is blindfolded. Then, the Participant starts reading off the cards in order, from the top of the disordered pile, telling the Magician what card it is. The Magician is able to guess whether the card is face up or face down, with nearly flawless accuracy (much better than 5 correct—the expected number due to random guessing)!

   (c) *Royal Flush Hummer*. The Magician takes about half a deck and shows the cards in it to the Participant, who is invited to shuffle them. The magician then apparently messes the cards up further in a random way with respect to orientation (face-up vs. face-down). Then the Magician invites the Participant to continue messing up the cards with some Hummer-type shuffles. Then the Magician deals the cards into two piles, puts them together, and spreads them out. Exactly 5 cards are face-down. They miraculously form a royal flush!

9 *Scarne's Lie Speller*. The Magician spreads out cards from the top of a deck and invites the Participant to pick one. He does so, notes the card, and puts it on the top of the deck, without the Magician seeing it. The Magician then cuts the cards, and puts them behind her back. She says, "I will flip over a random card and put it in deck somewhere."

Then she spreads the cards out from the top, stopping at the one face-up card. She says that if this card is red, the Participant must tell the truth, but if it is black, then the Participant may lie. The Magician sets aside the cards that were above the face-up card, and holds the cards below the face-up card. She then asks the following questions:

1. "Is the card red or black?" Depending on the answer, she deals out that many cards (e.g., if the answer is "red," she deals "R-E-D" from her pile (the cards below the face-up card).

2. "Is the card seven, above seven, or below seven?" Again, she deals out the answer.

3. "Is the card hearts or diamonds/ clubs or spades?" (depending on the answer to #1).

When she deals the answer to this question, the chosen card magically appears!

10 *Random Numbers*. The Magician asks the Participant to choose a random number n between 1 and 20, and share this number with the audience without letting the Magician know. The Participant then removes the top n cards from the deck.

Next, the Magician deals 20 cards from the top of the diminished deck (which is missing n cards), and he asks the audience to notice the nth card dealt (without giving it away with body language!).

Next, an audience member is asked to estimate half the size of the now very diminished deck (it is missing $20 + n$ cards). We call this number h. The Magician then deals h cards from the top, face-down. Then he places the stack of 20 cards on top of this, and puts the rest of the diminished deck on top of that (so the n cards removed at the start are still missing).

Finally, the Magician deals cards off the top, but at some miraculous point, stops, and it is the one that the audience noted!

11 *Guess from the Cut*. The Magician hands a deck of cards to several Participants, and asks one to cut the cards. Then she asks each of three Participants to draw from the top of the deck. She asks the first Participant to say the value of his card (not the suit), and asks the second to merely state his suit. For the third person, she only asks that the participant concentrate mentally on broadcasting his card. Of course, the Magician is able to correctly identify all three cards!