A story about Equivalence:
How I knew where Evans 330 was and that on March 6, 2012, I will not see the sunset.

One set; several equivalence relations

1. {752, 753, 852, 853}
   For example: Think of numbers and their remainders upon division by 2 or 3, house numbers, office numbers in Evans Hall.

2. People in this room:
   Show a color based on your row/column/diagonal distance from the person marked as "center."

3. Objects in envelope 1:
   {{0, 2, 4, 6, 8}, {1, 3, 5, 7, 9}}
   {{0, 3, 6, 9}, {1, 4, 7}, {2, 5, 8}}
   {{0, 4, 6, 9}, {1, 2, 3, 5, 7}, {8}}
   {{0, 4, 5, 9}, {1, 2, 6}, {3, 7, 8}}

What is the shape of "Equivalence"?

4. Points in the plane:
   Using the colors at our disposal (linked to 0, 1-10, 12, "every other number"), color each point in the plane according to its distance from the special point "0."
   What are the shapes that you see?

5. Ambitious Attempt #1
   _____

A checklist for equivalence
Ingredients:
- 1 Set
-
-
-

Investigating an Equivalence Relation
Did you find an equivalence relation on the set?

Tell me more. Answer as many of these as possible:

- What are the equivalence classes?
  What shape do they have?

- How many classes are there? ("A lot" is a code name for "Infinitely many")

- What is the relation?
  Ambitious question #1
  _____
  _____ ?

I have a problem!
The many ways I will use "Equivalence" to solve it.

On the right are some examples of functions we encountered and will encounter.

We spoke of the set up for a function:

The **domain**

The **co-domain**

Certainty
Pick one element from the domain.
The function will accept the element you picked and it will bring back a specific element from the co-domain.

For the **Lights** function
Domain: {positions of the classroom light switch}
Co-domain: {room lights on, room lights off}

For the **Dial** function, we suggested
Domain: {buttons on my phone, pressed once}
Co-domain: {commands my phone performs}

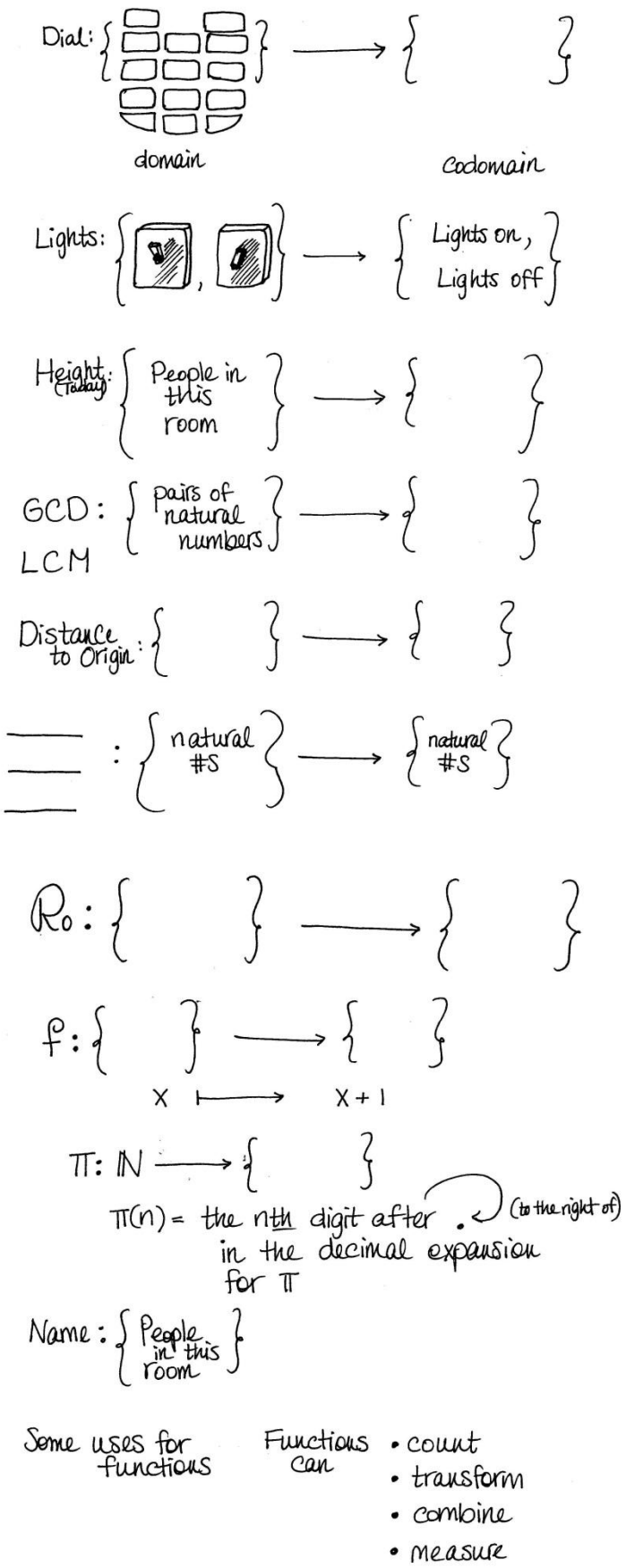
We saw 2 different functions that had the same domain and the same co-domain:

For instance, do you remember the 2 different functions with—

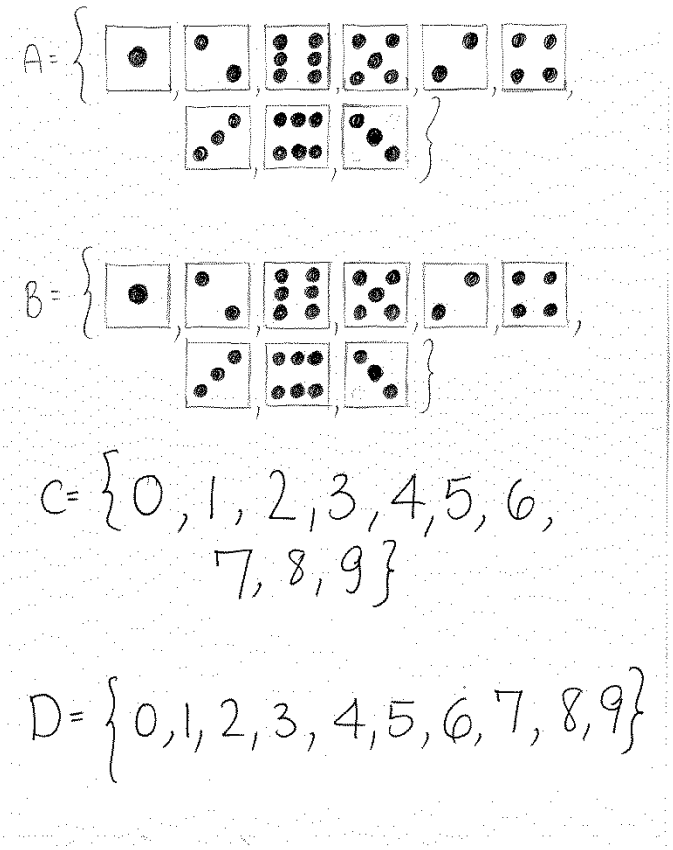
Domain: {objects that resembled faces of a die}
Co-domain: {1, 2, 3, 4, 5, 6}?

The function needs to know what to do with every single element of the domain.

The co-domain can contain elements the function never actually returns.



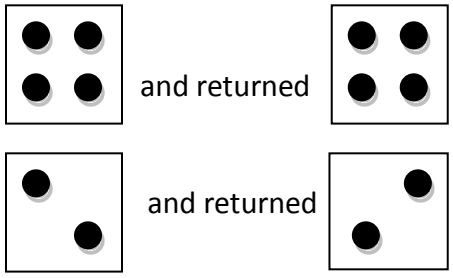
Remember some functions with…



<u>Set A as Domain, and Set B as Co-domain?</u>
We find the **Ro** functions, dedicated to Ro, the Math Circles teacher:

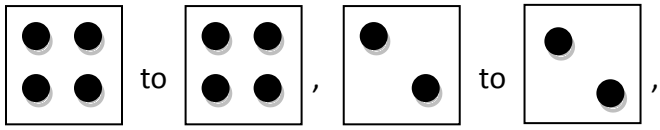
These are counter clockwise **Ro(tations)** of the plane. (Rotations that are in the opposite direction of how we usually see the hands of a clock move.).

Ro(tation) by 90°, took

 and returned ,

 and returned , and so on…

Ro(tation) by 180° surprisingly took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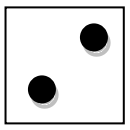
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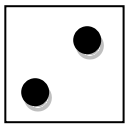 , and so on.

<u>Set C as Domain and Set D as Co-domain?</u>
I suggested we use some of the equivalence relations in the "*Idea #1 – Equivalence*" sessions.
The following functions remind us:
The definition of **function** does not include a requirement to use every element of the co-domain.

The **Count Holes** function:
This function sees Set C as a set of shapes. It counts the holes in each shape:

$0 \mapsto 1$ (The function takes the shape 0 and returns 1)
$1 \mapsto 0, 2 \mapsto 0, 3 \mapsto 0, 4 \mapsto 1, 5 \mapsto 0$, and so on…

The **Remainder mod 2** function:
This function brings back 0 or 1, based on whether the number we enter is even or odd,
$0 \mapsto 0, 1 \mapsto 1, 2 \mapsto 0, 3 \mapsto 1$, and so on …