

Math Dash
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The Dash: Instructions. 3 points will be awarded for the first correct answer to each question; 2 points will be awarded for the second correct answer to each question; 1 point will be awarded for the third correct answer to each question. Write your team letter, the problem number, and your answer (in simplest form) on your slip of paper before leaving your seat. You may not work on questions while waiting to show your answer to the judges. Thus, you should bring only your solution slip with you when you dash to the front. You may bring only one solution slip at a time. Teams are allowed to resubmit an answer if their first answer is correct, but must get back in line to do so. No calculators are allowed—pen/pencil and paper only! Good luck, and have fun!

PLEASE DO NOT OPEN OR TURN OVER UNTIL THE DASH BEGINS!

1. How many positive factors of 36 are also multiples of 4?
2. How many integers between 1000 and 2000 have all three of the numbers 15, 20, and 25 as factors?
3. How many three-digit numbers are divisible by 13?
4. Find the smallest positive integer that has a remainder of 4 when divided by 6 and a remainder of 3 when divided by 5.
5. The sum of the digits of a 2-digit number is subtracted from the number. The units digit of the result is 6. How many 2-digit numbers have this property?
6. How many 4-digit numbers with the 2 middle digits 97 are divisible by 45? What are the numbers?
7. How many 4-digit numbers with the 2 middle digits 38 are divisible by 11? What are the numbers?
8. The number 10101_2 is written in base 2. What is its representation in ordinary base 10?
9. The number 1000 is written in ordinary base 10. How is it written once it's converted to base 7?
10. An alien blackboard bears a half-erased calculation exercise:

$$\begin{array}{r} 2 \ 3 \ ? \ 5 \ ? \\ + \ 1 \ ? \ 6 \ 4 \ 2 \\ \hline 4 \ 2 \ 4 \ 2 \ 3 \end{array}$$

What number base was used?

11. A spaceship full of hostile aliens is about to land on Earth. The aliens are very fond of Earth donuts, and you have persuaded them to leave you in peace in exchange for one donut for each alien on board. The captain radios down and says: "There are 100 of us total on board, and we would like 24 jelly donuts and 32 chocolate donuts with sprinkles." What number system is he using?
12. Add $11121_3 + 120110_3$ (in base 3). Give your answer in base 3.
13. Multiply 102_3 by 201_3 (in base 3). Give your answer in base 3.
14. One digit of a number written in base 5 has been erased. The remaining digits are 4, 2, 0, 1, 3, 3, 2. If we know that the original number is even (divisible by 2), what are the possible values for the missing digit? Remember, the number is written in base 5.
15. How many zeros are at the end of the number $100!$?
16. Find the smallest natural number n such that $n!$ is divisible by 990.
17. What is the greatest common divisor of 312 and 180?
18. What is the greatest common divisor of 451 and 287?
19. How many two-digit numbers have digits whose sum is a perfect square?
20. For how many positive integer values of n are both $\frac{n}{3}$ and $3n$ three-digit whole numbers?
21. What is the last digit of the number 2012^{2012} ?
22. What is the remainder when 3^{2012} is divided by 7?
23. A standard 6-sided die is rolled, and P is the product of the 5 numbers that are visible. What is the largest number that must divide P ?
24. What is the sum of the digits of the product $2^{1999} \cdot 5^{2001}$?
25. For how many positive integers n less than or equal to 200 is $n^3 + 2n$ divisible by 3?
26. Find the last digit of the number $1^2 + 2^2 + 3^2 + \dots + 99^2$.
27. The number $25^{64} \cdot 64^{25}$ is the square of a positive integer N . What is the sum of the digits of N ?
28. For any integer n greater than 1, how many prime numbers are there greater than $n! + 1$ and less than $n! + n$?
29. How many positive integers are factors of 200?
30. Find the remainder when the number 9^{100} is divided by 8.
31. Find the remainder when the number $1989 \cdot 1990 \cdot 1991 + 1992^3$ is divided by 7.
32. There are 24 four-digit numbers that use each of the four digits 2, 4, 5, 7 exactly once. Only one of these four-digit numbers is a multiple of another one. What is it?
33. There are **200** light switches on the wall, all turned off. Two hundred kids come by. The first kid flips every switch. Then the second kid flips just switches 2, 4, 6, 8, ... etc. Then the third kid flips switches 3, 6, 9, 12, ... etc. This pattern continues until finally the 200th kid flips just switch number 200. How many lights are turned on at the end?

Most of these problems come from the AMC-8 contest, the AMC-10 contest, the AMC-12 contest, and the book *Mathematical Circles: The Russian Experience*.