Euclid's Game

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1 Number of Factors

1. How many factors (divisors) does the number 15 have (including 1 and itself)?
2. How many factors does the number 100 have?
3. More generally, if p, q, and r are three different prime numbers, find the number of different factors of:
   (a) pq
   (b) p^2q
   (c) p^2q^2
   (d) p^kq^m
   (e) $p^kq^mr^n$

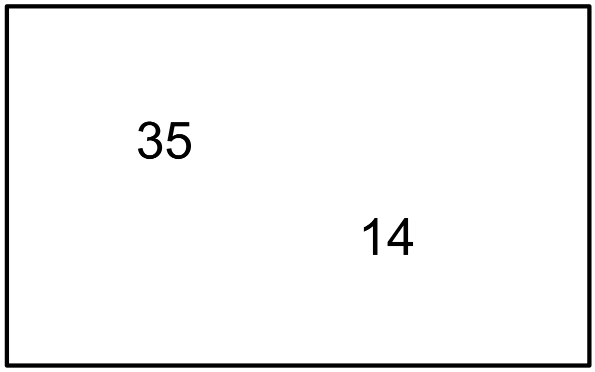
2 Greatest Common Factor and Least Common Multiple

- The **Greatest Common Divisor (gcd)** of two natural numbers is the greatest natural number that divides them both.
- The **Least Common Multiple (lcm)** of two natural numbers is the least natural number that is divisible by both of them.

4. Given numbers $x = 2^8 \cdot 5^3 \cdot 7$ and $y = 2^5 \cdot 3 \cdot 5^7$, find $gcd(x, y)$ and $lcm(x, y)$.
5. What is the gcd and the lcm of 2000 and 7200?
6. What is the gcd and the lcm of 847 and 539?
7. For how many values of k is 12^{12} the least common multiple of the natural numbers 6^6, 8^8 , and k?

3 Euclid's Game

See http://www.cut-the-knot.org/blue/EuclidAlg.shtml for an online version.



Start with two numbers in a box. Two players take turns writing a new number in the box that is the positive difference of two existing numbers in the box. The player that can no longer make a move wins.

8. Is there a winning strategy for Euclid's Game? Does it depend on what the two starting numbers are?
9. What is $gcd(949, 2701)$?
10. What is $gcd(451, 287)$?
11. Reduce the fraction $\frac{2023}{2431}$ to lowest terms.
12. Find the gcd of the numbers $2n + 13$ and $n + 7$.
13. Prove that the fraction $\frac{21n+4}{14n+3}$ cannot be reduced for any natural number n. (1959 IMO Problem 1)
14. Find $gcd(111...1111, 111.....111)$, where the first number has one hundred 1's and the second has sixty 1's.
15. Find $gcd(2^{100} - 1, 2^{120} - 1)$

Many of these problems are from *Mathematics Circles: the Russian Experience* by Fromkin, Genkin, and Itenberg