A Suite of Numerical Methods
from the practical to the esoteric

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Would you like to compute square-roots by hand? Know the digits of π to any number of places? Compute decimal values of fractions quickly? During this talk we will explore some methods of numerical computation, including "mental math" and several practical shortcuts. The exercises below are representative of problems solvable using techniques we will discuss. Solutions and hints will be provided during the talk.

Repeating Decimals

1. Find the rational number whose decimal expansion is .12357357357 …
2. Find the rational number whose decimal expansion is .12345678 …
3. Find the rational number whose decimal expansion is .010203040506 …
4. Find the rational number whose decimal expansion is .010204081632 ...
4. Find the rational number whose decimal expansion is .0103050709 ...
4. Find the rational number whose decimal expansion is .010407101316 ...

Fibonacci Numbers

1. Find the rational number whose decimal expansion contains the Fibonacci numbers: .01010203050813 ...
2. Repeat the previous problem with .00100100200300500801302 1 …

Fractions

1. Use the rapid method to compute several decimal places of $\frac{1}{17}$, $\frac{16}{19}$, $\frac{27}{29}$, $\frac{1}{49}$.

Other Infinite Series

As we saw, Euler solved the Basel Problem by showing that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

1. Find the value of $\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \cdots + \frac{1}{(2n)^2} + \cdots$
2. Prove that $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$.
3. Prove that $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \frac{1}{9^2} + \cdots + \frac{1}{(2n+1)^2} + \cdots = \frac{\pi^2}{8}$

Miscellaneous

1 What is the decimal expansion of $\frac{1}{(1-x)^2}$ at $x = .1$? At $x = .01$?

2 What is the decimal expansion of $\frac{1}{1-x-x^2}$ at $x = .1$? At $x = .01$?

Sources

Rapid Decimal Expansions..., by Alex Balfour (*The Mathematical Gazette*, Vol 87, No 509, July 2003)

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A Spigot Algorithm for the digits of π, Rabinowitz and Wagon, (*The American Mathematical Monthly*, 102(1995))

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