Berkeley Math Circle — Oct. 25, 2011

Games and Coloring

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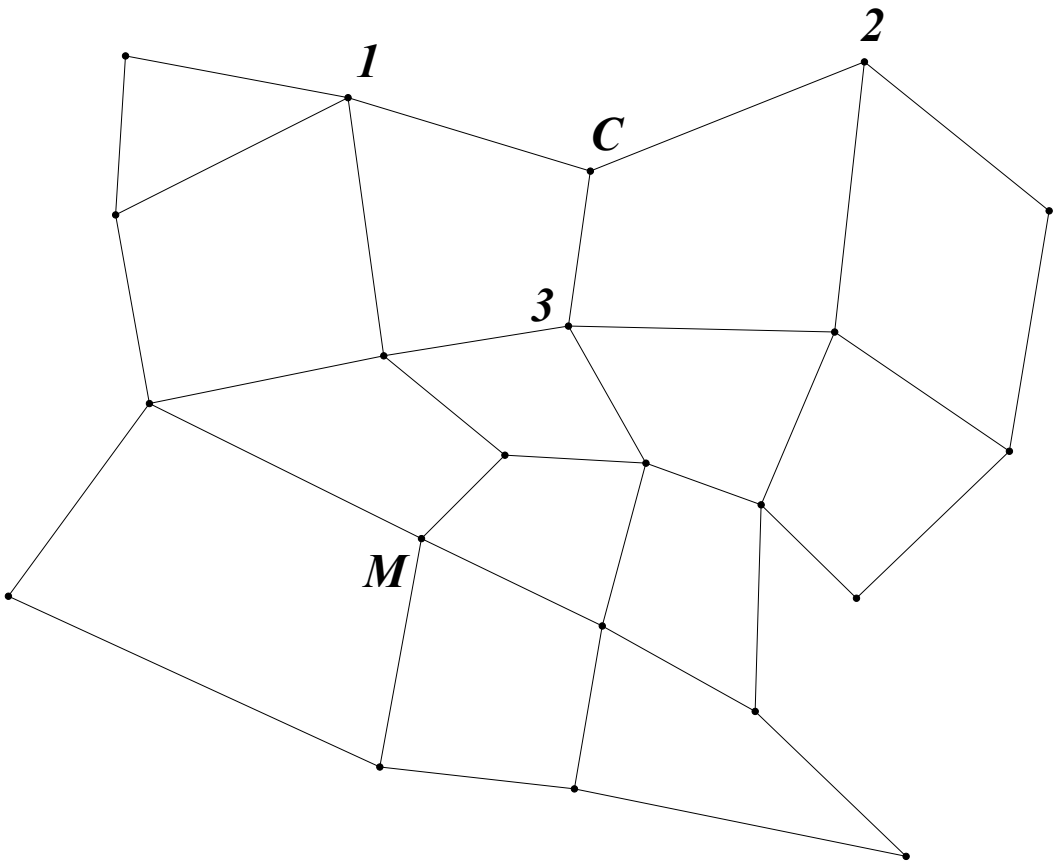
1 *Cat and Mouse*. A very polite cat chases an equally polite mouse. They take turns moving on the grid depicted below.



Initially, the cat is at the point labeled C; the mouse is at M. The cat goes first, and can move to any neighboring point connected to it by a single edge. Thus the cat can go to points 1, 2, or 3, but no others, on its first turn. The cat wins if it can reach the mouse in 15 or fewer moves. Can the cat win?

2 *Putdown*. Each player takes turns placing a penny on the surface of a rectangular table. No penny can touch a penny that is already on the table. The table starts out completely bare. The winner is the person who makes the last legal move. Can you find a strategy to win this game? What if the penny is replaced with a card (for example, from a deck of cards)?

3 *A Two-Colored World*. There are finitely many points in the plane, colored either blue or red. These points obey two laws:

1. Between any two red points, on the line segment joining them, there is at least one blue point.
2. Between any two blue points, on the line segment joining them, there is at least one red point.

Can you describe the types of possible worlds that obey these rules?

4 *Proper Colorings*. A map is *properly colored* if no adjacent regions have the same color.

(a) Suppose you draw a map just using straight (infinite) lines. How many colors do you need?

(b) What if you use circles instead of lines? How many colors do you need now?

5 *Gallery Guards*. A gallery is an enclosed room with 2001 walls. Guards are stationed to make sure no one steals any paintings. The guards cannot move, but they can swivel their heads around. What is the least number of guards needed to *guarantee* that all walls can be watched?

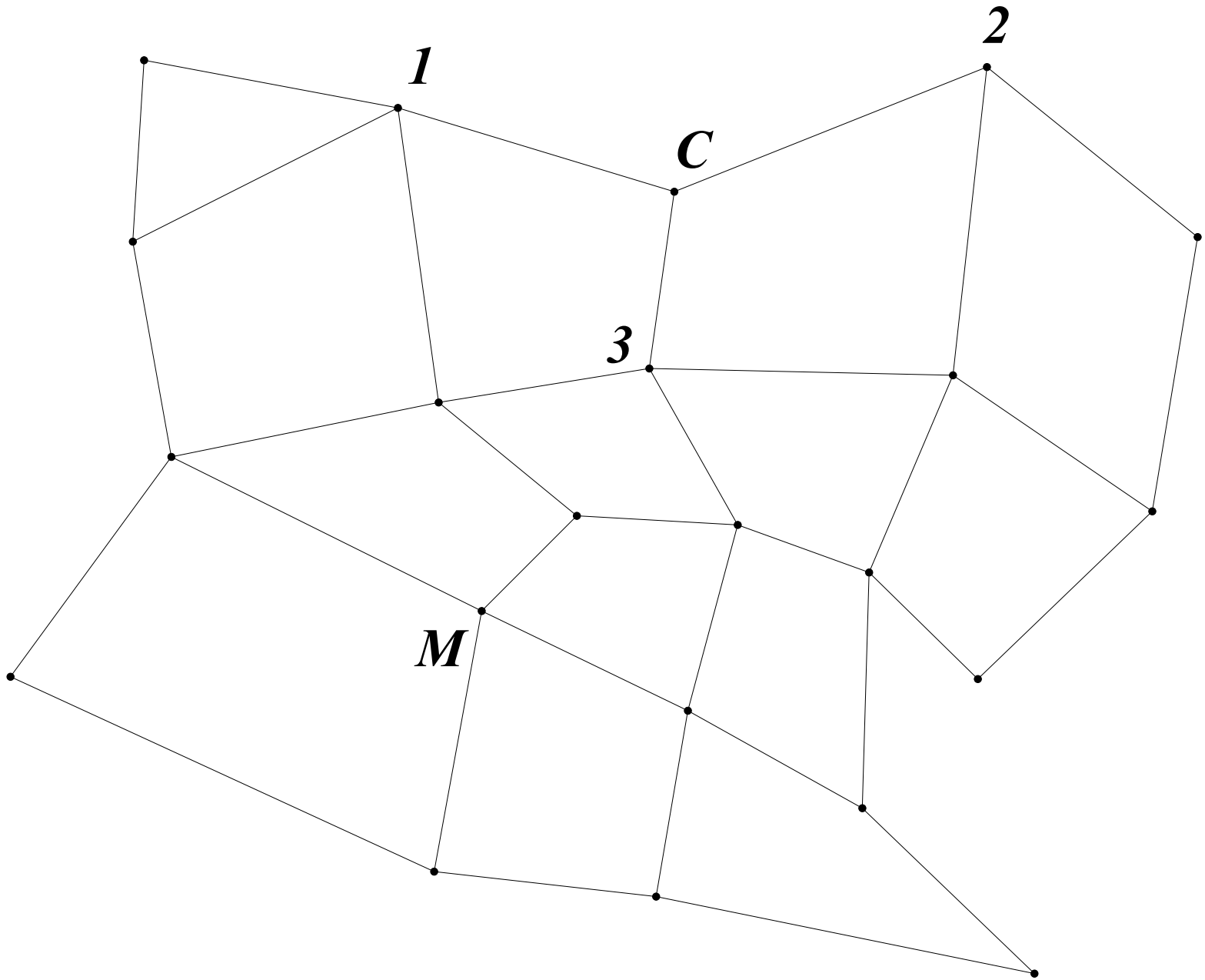
6 *Tiling Problems*. An geometric shape can be *tiled* if it can be covered with smaller objects so that it is completely covered, with no overlaps. For example, an 8×8 chessboard can be tiled by 2×1 "dominos," but cannot be tiled by 3×1 "long dominos." (why?)

(a) Can an 8×8 chessboard with the two opposite corners removed be tiled by 2×1 dominos?

(b) Can a 10×10 chessboard be tiled with 4×1 long dominos?

(c) Which rectangles can be tiled with "ells?" (An ell is a 2×2 square with one 1×1 square removed.)

(d) Can an 8×8 chessboard with a single square removed be tiled by ells?


1
2
C
3
M