Counting Fundamentals III: Counting, Adding, and Multiplying

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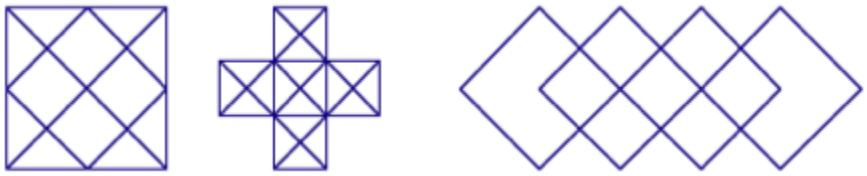
Problem 1. Arrange the numbers 1, 10, 100, 1000, ... in a 3 by 3 square, and look at the 8 possible sums: the three rows, the three columns, and the two diagonals. What is the smallest possible sum? What is the largest possible sum?

Problem 2. A very large container is filled with red, green, and purple candies in equal numbers, so that each color is equally likely. You grab a handful of three candies without looking and toss them in your mouth. How many different flavor experiences are possible? Which of them are most or least likely?

Problem 3. How many palindromes are between 12345 and 54321?

Problem 4. How many 4-digit PINs (thus including things like 0164 as a four-digit "number") have no two adjacent digits equal?

Problem 5. How many triangles are in each of the diagrams below? How many rectangles?



Problem 6. Write a standard 10x10 multiplication table. What is the sum of all 100 numbers?

Problem 7. How many squares are there on a chessboard? How many rectangles?

Problem 8. What is the largest number of regions in the plane that can be created by n lines?
What about n circles in the plane?
What about two n-gons in the plane?
What about n planes in space?

Problem 9. How many ways are there to win by getting 5 in a row in 19x19 tic-tac-toe? How about 4 in a row in 4x4x4 tic-tac-toe?