Counting Fundamentals I:
Counting, Adding, and Multiplying

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Problem 1. How many numbers are in the list 1, 2, 3, 4, 5?
How about 0, 1, 2, 3, 4, 5?
How about 0, 2, 4, 6, 8, 10?
How about 0, 2, 4, 6, 8, ..., 100?
How about 3, 6, 9, 12, ..., 279?
How about 10, 17, 24, ..., 276?
What is the sum of the numbers in each of these lists?

Problem 2. Two people share $200 in such a way that each person gets a whole number of dollars, and at least $25. How many ways are there to do this?
What if it's also true that neither person can get an amount divisible by $3?

Problem 3. A book has pages numbered 1 through 777. How many occurrences of each digit are there?

Problem 4. Arrange the numbers 1 through 9 in a square, and look at the 8 possible sums: the three rows, the three columns, and the two diagonals. What is the smallest possible sum? What is the largest possible sum?

What if we use the numbers 1, 3, 5, ..., (what should the last number be) instead?

Next week we'll try it with the numbers 1, 10, 100, ..., so if you get bored today, think about ways of approaching that.

Problem 5. Between 1am and 1pm, how many times is the sum of the hours and minutes digits on a digital clock exactly 5? What could we change about this problem to make it easier? Harder?

Problem 6. A very large container is filled with red, green, and purple candies in equal numbers, so that each color is equally likely. You grab a handful of three candies without looking and toss them in your mouth. How many different flavor experiences are possible? Which of them are most or least likely?

Problem 7. How many palindromes are between 12345 and 54321?

Problem 8. How many 4-digit PINs (thus including things like 0164 as a four-digit "number") have no two adjacent digits equal?

Problem 9. How many triangles are in each of the diagrams below? How many rectangles?