Counting Fundamentals IV: Counting, Adding, and Multiplying

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Many of these problems are from the AMC8, perhaps with some slight modifications, and are noted by the year in which they appeared.

Problem 1. A *composition* is a way of writing a number as the sum of other numbers. For example, compositions of 4 include 4, 1+1+2, and 1+2+1. How many compositions does 4 have? What happens if we look at numbers other than 4? What if we then organize them by how many parts they have? What if we organize them by how big the largest part is?

Problem 2. A *partition* of a number is like a composition, but order doesn't matter, so 1+1+2 and 1+2+1 are two different ways to write the same partition of 4. How many partitions does 4 have?

Problem 3. How many ways are there to make change for a dollar?

Problem 4. (2007) A haunted house has six windows. In how many ways can Georgie the Ghost enter the house by one window and leave by a different window?

Problem 5. (2004) How many different four-digit numbers can be formed by rearranging the four digits in 2004?

Problem 6. (2004) How many two-digit positive integers have digits that total 7?

Problem 7. (2005) How many three-digit numbers are divisible by 13?

Problem 8. (2002) How many whole numbers between 99 and 999 contain exactly one 0?

Problem 9. (2000) Assuming every president serves a full four-year term, how many presidents could we have in the span of years from 2013 through 2025?

Problem 10. (2007) Two cards are dealt from a deck of four red cards labeled A, B, C, D and four green cards labeled A, B, C, D. A winning pair is two of the same color or two of the same letter. What is the probability of drawing a winning pair?

Problem 11. (2007) A bag contains four pieces of paper, labeled 1, 2, 3, and 4. By drawing pieces of paper one at a time, a three-digit number is formed. What is the probability that the number is a multiple of 3?

Problem 12. (2002) What is the probability that in four tosses of a fair coin, you get at least as many heads as tails?

Problem 13. (2006) A tennis tournament has six players, with each player playing every other player once. If Helen won 4 games, Ines won 3 games, Janet won 2 games, Kendra won 2 games, and Lara won 2 games, how many games did Monica win?

Problem 14. (1998) What are all possible values for the sum of the numbers showing on four six-sided dice if the product is 144?

Problem 15. (2005) How many different isosceles triangles have integer side lengths and a perimeter of 23?
How about a perimeter of 24?
How about a perimeter of 2005?

Problem 16. (2005) The Little Twelve Basketball Conference has two divisions with six teams in each division. Each team plays each other team in its division twice and each team in the other division once. How many games in total are played?

Problem 17. (2004) Three friends have a total of 6 identical pencils, with each having at least one pencil. How many ways are there for this to happen?

Problem 18. (1996) How many subsets containing three different numbers from the set $\{89, 95, 99, 132, 166, 173\}$have an even sum?

Problem 19. How many triangles are in an n by n triangular grid?

Problem 20. (2000) Zvezda flips 1 coin and Paul flips 2 coins. What is the probability that they have the same number of heads?

Problem 21. (1998) A phone number is "memorable" if the first three digits are the same as the next three digits, as in 456-4568, or as the last three digits, as in 456-8456, or possibly both. Assuming that all seven digits can be any value 0 through 9, how many different memorable phone numbers are there?