Combinatorics - V

Berkeley Math Circle - Beginner's

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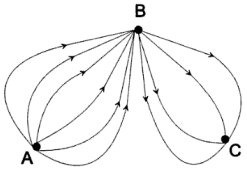
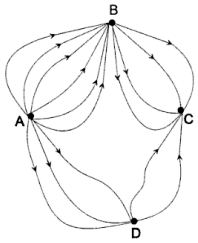
Many of these problems are from *Mathematical Circles (Russian Experience)* and from *A Decade of the Berkeley Math Circle -Volume 1*

1. A taqueria sells burritos with the following fillings: pork, chicken, beef, and tofu. Burritos come either small, medium, or large, with or without cheese, and with or without guacamole. How many different burritos can be ordered?

2. There are three towns A, B, C, on Frog Island. There are 6 roads from A to B and 4 roads from B to C. How many ways can you drive from A to C (without going back through A!)



3. A new town D is built and 5 new roads as shown. Now how many ways are there to drive from A to C?



Multiplication vs. Addition:

a) If we are counting the number of outcomes of a multi-stage process (like choosing from a sequence of menus), then the number of outcomes is the product of the number of choices for each stage.

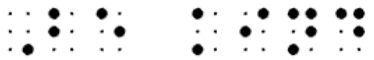
b) Whenever we separate the outcomes into several cases, each requiring separate counting methods, we add the number of outcomes in each case to get the total number of outcomes.

c) Sometimes its easiest to count the number of outcomes we don't want and subtract that number from the total number of outcomes.

4. There are 11 girls and 17 boys in Ms. Jewls' class. How many ways are there to choose one boy and one girl to represent the class in a math contest? How many ways are there to choose 2 students of the same gender?

5. There are 5 books on a shelf. How many ways are there to arrange some or all of them in a stack? A stack may contain only one book.

6. The Braille writing system uses 6 dots to represent the 26 letters of the English alphabet. Are 6 dots also enough to represent the Japanese Katakana syllabic writing system, with 46 symbols?



7. I have 3 kids and a husband. In how many ways can I choose one or more family members to accompany me on a hike?

8. You roll a 6-sided die 3 times. Among all possible outcomes, how many have at least one occurrence of the number 6?

9. How many five digit numbers have an even sum of their digits?

10. You flip a coin 10 times. Of all the possible outcomes, how many have exactly 5 heads in a row? For example, we would not count HHHHHHHTTT (too many consecutive heads), but we would count *TTTHHHHHTT* and *HHTTHHHHHT*.

11. How many even three digit numbers have no repeating digits?

12. Seven children are lining up for recess. In how many different orders can they line up?

13. How many ways are there to arrange the letters in the word SWEETHEART?

14. All 11 girls in Ms. Jewls' class are going to the circus, and will be sitting in a row of 11 seats. How many ways are there for them to seat themselves if Mauricia and Deedee refuse to sit next to each other?

15. There are 6 boys and 6 girls in a dance class. In how many ways can they partner off into boy-girl couples?

16. How many ways are their to seat 10 girls and 10 boys in a row if all boys must sit together and all girls must sit together? What if boys and girls must alternate?

Permutations:
Permutations: The number of ways to lay out n different objects in a row is $n! = n \cdot (n-1) \cdot (n-2) \ldots 3 \cdot 2 \cdot 1$.

For next time:

A. What is the largest number of triangles you can make by drawing 7 lines in the plane? The triangles may overlap or contain each other.

B. How many ways are there to represent the number 12 as a sum of

   (a) 5 non-negative integers?

   (b) 5 positive integers?

   The order of the numbers matters here, so, for example, 1 + 4 + 5 + 1 + 1 is considered different from 1 + 1 + 1 + 4 + 5.