PROBLEMS ON SYSTEMS OF GEOMETRICAL STRUCTURES
MARCH 29TH, 2011

Problem 1: Let A, B, C, and D be points on the plane with two continuous paths connecting A to C and B to D. Moreover, it is known that two points X and Y can start at A and B, respectively, and move along the paths ending at C and D so that the distance between them is always less than or equal to 2. Is it ever possible to move two disks S_1 and S_2 of radius 1 along the paths starting at A and D and ending at C and B so that they never touch each other and their centers always stay on the paths?

Problem 2: Consider a circle C of radius 1 that contains some number of circles the sum of whose diameters is less than 1. Show that it is possible to draw a circle concentric with C that does not touch any of the internal circles.

Problem 3: There are $n \geq 3$ points on the plane and not all of them are collinear. Show that there exists a circle passing through 3 of the given points and not containing any other given point.

Problem 4: There are $n \geq 3$ points on the plane with all pairwise distances being different. Each point is connected with a segment to the closest point. Can one get a closed broken line?

Problem 5: All points on the plane are colored one of three colors. Show that there exist two points of the same color with distance 1 between them.

Problem 6: There are a finite number of points on the plane such that any line passing through two of them contains another given point. Show that all given points are collinear.

Problem 7: There is a finite number of (not necessarily convex) n-gons on a plane such that any two of them intersect. Show that there exists a line that intersects all of the given n-gons.

Problem 8: There n red and n blue points on the plane, no three of which are collinear. Show that it is possible to draw n segments between points of different colors so that no two of them intersect.