Voting Schemes: is democracy possible?

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When five people need to choose between two options X and Y everybody knows what to do: they vote.

Exercise. Six people need to choose between two options X and Y. Can you invent a perfectly fair way of resolving draws (no voter and no option must get any advantage)?

With two options the draw is the only theoretical problem. With a large number of voters it is not likely to occur in real life. Because of that no democratic voting system explains what to do in case of a draw as we saw in Florida during the 2000 Presidential Election: Bush won by 537 votes (0.009%), which is equivalent to a draw for all practical purposes.

The situation is much worse with 3 or more options.

Example. A group of 12 professors was given a choice between three possible postdocs X, Y, and Z. They ranked them as follows:
- 5 ballots read $Y > Z > X$,
- 4 ballots read $Z > X > Y$,
- 3 ballots read $X > Y > Z$.

Who do you think should be chosen? What if Z says that finally he found a position elsewhere?

Exercise. Three runners X, Y, and Z are running a semi-marathon. As a promoter I have enough money to hire only one of them. If my runner wins the race I get a price of 1 million dollars. I know that Y is a very reliable runner who always runs the distance in 1h02. The other two runners, however, can be in good shape or not. X has 2 chances out of five to be in good shape, in which case he covers the distance in 1h00. Otherwise he makes it in 1h03. Z has 1 chance out of two to be in good shape, in which case he covers the distance in 1h01. Otherwise he makes it in 1h04. Which runner should I hire? What if Z breaks his leg and cannot participate?

The two examples above illustrate the "spoiler effect" a third candidate does not win the ballot, but ruins the victory for another candidate. Do you think it is acceptable to permit the spoiler effect in a fair voting system?

Arrow's theorem. Suppose N voters vote for at least three candidates. A bollot consists of a ranking of all candidates in the order of preference. We wish to create a system, called a voting scheme, that converts the ballots into one ranking of the candidates relfecting the preferences of the group. We would like the voting scheme to satisfy the following conditions:

- If every voter prefers X to Y then the group prefers X to Y.
- The outcome between X and Y depends only on how the voters ordered X and Y; not on their preferences for any other candidates.
- The voting scheme should be deterministic: no lot drawing is allowed.

Then the only possible voting scheme is a dictatorship; in other words it consists in adopting the ranking of one of the voters chosen in advance.

We are going to prove this theorem during the session.