Examples of Dysfunctional Functions (aka Relations)

Below is a list of relations on various sets. Determine which of these relations are functions. Further try to find all of the properties of the given relations (reflexivity, irreflexivity, symmetry, anti-symmetry, transititvity).

1) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $a - b$ is negative.

2) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $a - b$ is nonnegative.

3) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $a - b$ is rational.

4) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $|a| = b$.

5) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $|a| = |b|$.

6) Let R be the relation on the integers defined by $(a, b) \in R$ whenever $a - b$ is even.

7) Let R be the relation on the reals defined by $(a, b) \in R$ whenever $|a - b| < 1$.

8) Let R be the relation on the positive integers defined by $(a, b) \in R$ whenever a divides b (ie a goes evenly into b).

9) Let R be the relation on the integers where for every pair (a, b) we have $(a, b) \in R$. (This is called the total relation).

10) Let R be the relation on the integers where for every pair (a, b) we have $(a, b) \notin R$. (This is called the empty relation).

11) Let R be the empty relation on the empty set.

12) Let R be the relation on the set of all people defined by $(a, b) \in R$ whenever a has exactly the same first name as b.

13) Let R be the relation on the set of all people defined by $(a, b) \in R$ whenever a speaks at least one language in common with b.

14) Let R be the relation on the set of all people defined by $(a, b) \in R$ whenever a is in love with b.

15) Let R be the relation on the set of all people defined by $(a, b) \in R$ whenever a has biological father b.

16) Let R be the relation on the set of all people defined by $(a, b) \in R$ whenever a is the sister of b.

17) Let R be the relation on the set of all women defined by $(a, b) \in R$ whenever a is the sister of b.