Definitions for Dysfunctional Functions (aka Relations)

A **relation** R from a set A to a set B is a collection of ordered pairs whose first entry belongs to A and whose second entry belongs to B. If we take A and B to be the same we say R is a relation on A.

The **domain** of a function is the set of all points $a \in A$ for which there exists a $b \in B$ such that $(a, b) \in R$.

$$\mathrm{Dom}(R) = \{a \in A \mid \exists b \in B \text{ s.t. } (a, b) \in R\}$$

The **range** of a function is the set of all points $b \in B$ for which there exists a $a \in A$ such that $(a, b) \in R$.

$$\mathrm{Ran}(R) = \{b \in B \mid \exists a \in A \text{ s.t. } (a, b) \in R\}$$

A **function** F from A to B is a relation with the property that for each element a in the domain there exists a unique element b in the range such that $(a, b) \in R$.
(i.e. If (a, b) and (a, c) belong to the relation, then $b = c$.)

A relation R on A is **reflexive** if for each element $a \in A$ we have $(a, a) \in R$.
The relation is called **irreflexive** if for each element $a \in A$ we have $(a, a) \notin R$.

A relation R on A is **symmetric** if whenever $(a, b) \in R$ we also have $(b, a) \in R$.
The relation is **anti-symmetric** if whenever (a, b) and (b, a) belong to R then $a = b$.

We say that a relation R on A is **transitive** if whenever (a, b) and (b, c) belong to R, then (a, c) is also in R.

A **partition** on a set A is a collection $\mathcal{P}$ of subsets of A with the following properties

$$\begin{aligned}
&(1)\ \ A = \bigcup\{X \mid X \in \mathcal{P}\} \\
&(2)\ \ X \neq \emptyset \quad \forall X \in \mathcal{P} \\
&(3)\ \ X \cap Y = \emptyset \text{ whenever } X, Y \in \mathcal{P} \text{ with } X \neq Y
\end{aligned}$$

An **equivalence relation** is a relation that is reflexive, symmetric, and transitive.
If R is an equivalence relation on A, we will often write $a \sim b$ to denote $(a, b) \in R$.
In this case we say a is equivalent to b.

Given an equivalence relation on A, we define the **equivalence relation** $[a]$ on an element a to be the set of all points in A which are equivalent to a.

$$[a] = \{b \in A \mid a \sim b\}$$