Mathematical games for two players.

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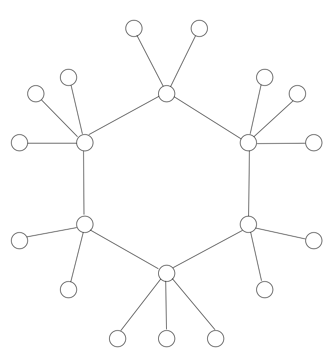
For each game, determine whether the first or the second player will win if both play in the best possible way. When possible describe the winning strategy for this player.

1. The first player tells a number between 1 and 10; the second player adds to it another number between 1 and 10; the first player adds to the sum another number between 1 and 10; and so on. The player who gets the sum 100 wins the game.

2. At the beginning of the game there are three piles of stones: one with 4 stones, one with 5 stones, and one with 6 stones. In one move every player can take any number of stones from any pile. The player who takes the last stone wins.

3. Two armies wage a war on the graph in the picture. At the beginning the first army parachutes its troops on one of the vertices. Then the second army parachutes its troops on another vertex (it cannot go on the same). In one move each army can occupy a free vertex that is connected by an edge to one of the vertices that it already holds. They do that in turns. If one of the armies has run out of accessible vertices, the other army still continues to play. Once all the vertices have been occupied, the army that holds more vertices than the other is declared the winner.



4. Two players take turns in placing 1×3 rectangles on a 1×20 checkered strip without overlapping. The player who cannot make a legal move loses.

5. At the beginning we have a pile with 100 stones. The first player can take any number of stones, but not all of them. After that each player can take

a) at most as many stones as the other player took on his previous move.

b) at most twice as many stones as the other player took on his previous move.

The player who takes the last stone wins.

6. Two players take turns in writing couples of nonnegative integers (n, m) on the blackboard. If a couple (n, m) is already on the blackboard, all couples (N, M) such that $N \geq n$, $M \geq m$ are forbidden. The player who has to write the couple $(0, 0, 0)$ loses.

6 bis. Two players take turns in writing triples of nonnegative integers (n, m, p) on the blackboard. If a triple (n, m, p) is already on the blackboard, all triples (N, M, P) such that $N \geq n$, $M \geq m$, $P \geq p$ are forbidden. The player who has to write the triple $(0, 0)$ loses. As far as I know this is an open problem.

7. Two players take turns in emitting coins; the value of each coin is an integer number of cents. It is forbidden to emit a 1-cent coin or a coin whose value you can make with the coins that have already been emitted. The player who cannot emit a coin loses. (E.g., suppose the first player emits a coin of 2 cents. Then the second player emits a coin of 3 cents and wins, because you can make any amount of cents greater then one using the 2- and 3-cent coins.)