FACTS and TOPICS Used, Proved or Touched Upon
in the Geometry series of sessions at BMC-beginners
September 2010
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From Geometry in the plane:

1. Any two *right* angles are equal (congruent).
2. *Vertical* angles are equal (congruent).
3. Criteria for *congruent triangles*:
   - SAS (side-angle-side)
   - SSS (side-side-side)
   - ASA (angle-side-angle)
   - Investigate to find a 4th criterion
4. Criteria for *similar triangles*:
   - AA (two angles)
   - SAS (one angle and two side ratios)
   - SSS (three side ratios)
   - Investigate to find a 4th criterion
5. Any two *isosceles right triangles* are similar, with 45-45-90 angles.
6. *Reflection* across a line
   - how to draw reflections of points and segments with a ruler and a compass?
   - reflection preserves segment lengths and angle measures.
7. *Pythagorean theorem*
8. *Triangle inequality* and shortest distance between two points
9. *Extra constructions* in a geometry problem are hard to come up with, but a suitable such construction may provide the most beautiful solution to the problem.
10. *Reflections across a line* often inspire extra constructions in geometry problems.

From Logic and Proofs:

11. Physical experiments help us come up with conjectures, but they do not constitute mathematical proofs.
12. Measuring geometric objects with tools (e.g., ruler, protractor, compass) is a physical experiment, and hence not a mathematical proof.
13. There may be more than one way to rigorously solve a mathematical problem.
14. Some problems may look easy, yet their rigorous solution may require several non-trivial steps.
15. Light travels the shortest possible route between objects , even when it must reflect off mirrors/mirror-like “walls”.

Farmer-and-Cow & 3-squares Problems

From Algebra:

16. Label unknown quantities by Latin letters (e.g., segment lengths) and by Greek letters (e.g., angles).
17. Construct equations that involve the unknowns.
18. Solve equation with fractions by cross-multiplying to get rid of all denominators.
19. Square roots satisfy certain rules. For instance, $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$ and $\sqrt{a^2} = a$ for positive a, b, c.

From Number Theory:

20. *Square roots* often produce complicated numbers:
    - whose decimal representations has no repeated pattern
    - which cannot be written as fractions of two whole numbers
    - and which are called *irrational* numbers.

Question: What to do in Geometry between now and the next 4-series Geometry session at BMC-beginners in the spring semester 2011?

Partial Anwer:

1. Buy and study Kiselev's Planimetry, Part I. (Copies can purchased through Professor Alexander Givental at UC Berkeley Math Dept, or through the web.)
2. Along with studying Kiselev's book, try to do some of the problems in the Geometry handout for BMC-beginners. You may have a hard time in the beginning with all or most problems on the handout; however, after studying Kiselev's book most of the unknown concepts and theory will be clarified.
3. Do the Monthly Contests at BMC. Read their solutions when they are distributed. Some of these problems will be in geometry. One such is already on the first Monthly Contest, due October 5, 2010.