Binary Arithmetic Worksheet

Convert the following numbers from binary to base 10:

1. $11_2 =$
2. $100_2 =$
3. $1001_2 =$
4. $1101_2 =$
5. $10101_2 =$

Convert the following numbers from base 10 to binary:

6. $11_{10} =$
7. $100_2 =$
8. $1001_2 =$
9. $1101_2 =$
10. $10101_2 =$

Counting in Binary

1	1
2	10
4	100
8	1000
16	10000
32	100000
64	1000000
128	10000000
256	100000000
512	1000000000
1024	10000000000

.
.
.

To convert a number from binary to base 10, just break the number into a sum of numbers from the above list.

Example: Convert 10011_2 to base 10

$$
\begin{array}{rr}
 & 10000_2 = 16 \\
+ & 10_2 = 2 \\
 & 1_2 = 1 \\
\hline
 & 19_{10}
\end{array}
$$

Or, written another way:

$10011_2 = 10000_2 + 10_2 + 1 = 16 + 2 + 1 = 19_{10}$

To convert a number from base 10 to binary, do the following:

1. Write down the largest binary number from the list which is less than or equal to the base 10 number.

2. Subtract the number from the base 10 number.

3. Repeat steps 1 and 2 with the new number.

Example: Convert 197_{10} to binary

1. The largest binary number that is no bigger than 197 is 128, which written in binary is 10000000.

2. Subtract 197 – 128 to get 69.

3. The largest binary number that is no bigger than 69 is 64, which in binary is 1000000.

4. Subtract 69 – 64 to get 5.

5. The largest binary number that is no bigger than 5 is 100.

6. Subtract 5 – 4 to get 1.

7. The largest binary number that is no bigger than 1 is 1 and we're done:

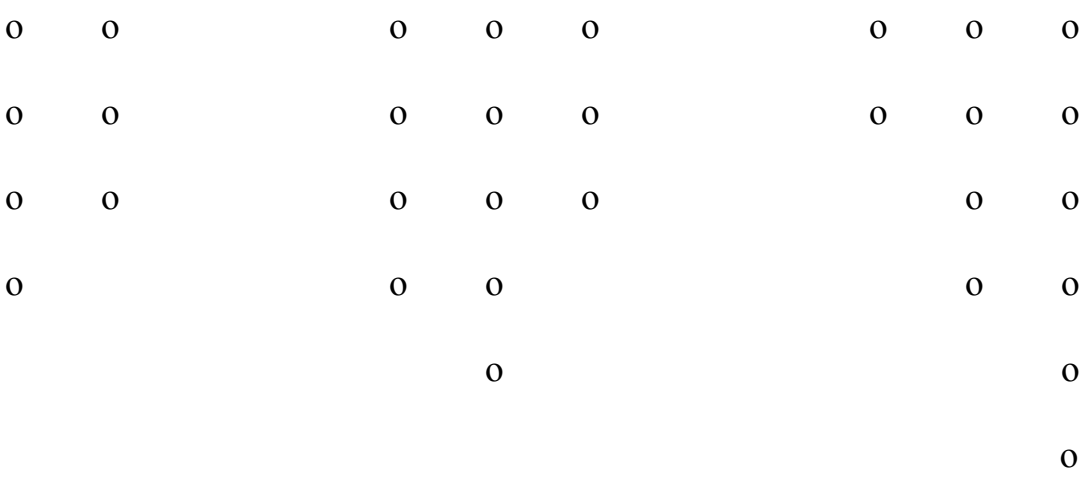
The binary number that is equivalent to 197_{10} is

$$
\begin{array}{rr}
 & 10000000_2 \\
+ & 1000000_2 \\
 & 100_2 \\
 & 1_2 \\
\hline
 & 11000101_2
\end{array}
$$

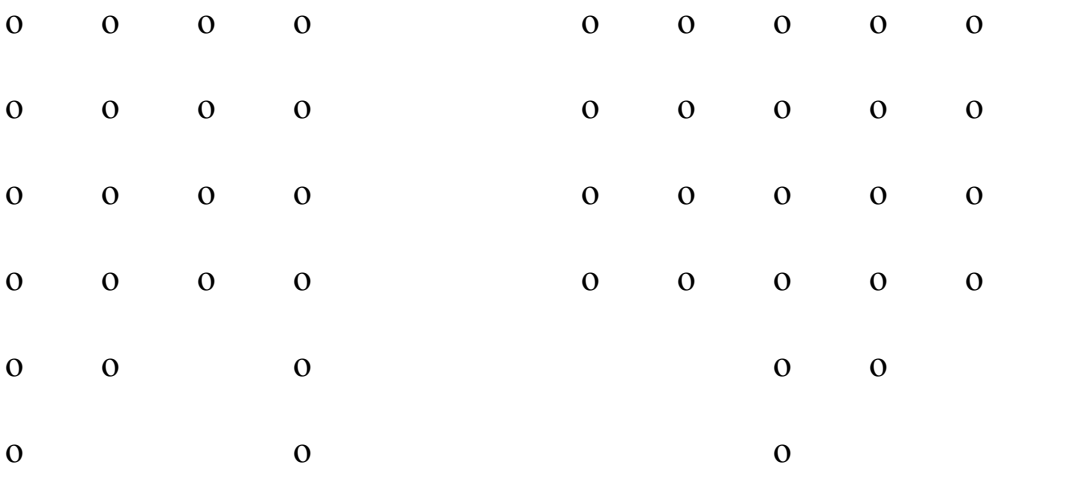
The Game of Nim

Object: Force your opponent to make the last move.

Rules: Two players each take turns crossing out circles. On a move, at least one circle must be crossed out and at most one entire column may be crossed out.



#1 #2 #3



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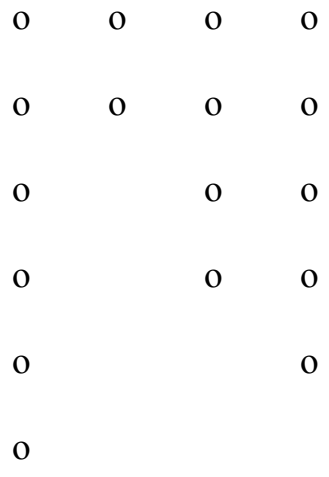

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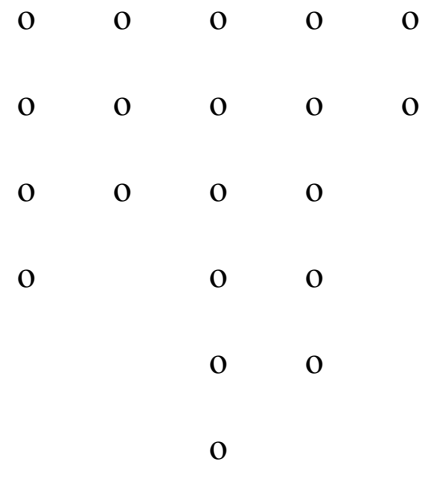

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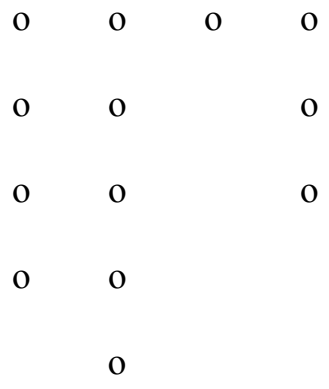

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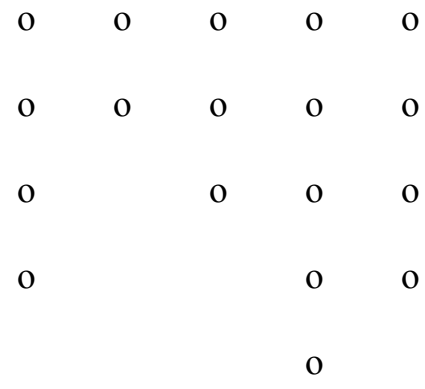

#9

How to Win at Nim

1. Write out the number of markers in each column in binary
2. Compute the Nim sum of all the columns and call it X
3. Take the Nim sum of X with each column and call it Y
4. Find a column whose size decreases
5. Remove enough markers so that what is left is Y

Example:

O O O
O O O
O O O
O O O
O O O
O O
O

111
110
101

X = 100

Compute the Nim sum of X with each column:

111
100

Y = 011

111
110

Y = 001

111

101

Y = 010

Solution: take away 4 (the value X) from any of the columns

Continue with the subsequent moves.