Lesson 10 November 10, 2009 BMC Elementary

Overview.
I was afraid that the problems that we were going to discuss on that lesson are too hard or too tiring for our participants. But it came out very well and we did a great job with both topics.

1. Our first topic was construction of negations (opposites) of given statements. As an introduction, we started with recalling how to say "yes" and "no" in different languages.

2. Given statements like *"All …. are …"* or *"Some… are not…"* we constructed the opposites (in formal logic) of those statements. As it was wisely noticed by one of the participants, the opposite for a statement is in some sense is like an antonym for a word. Since I did not give any definition of the formal opposite, the only way to understand the notion was through many examples. The easy and natural ones like

Some people can fly

All birds can fly

went very smoothly, but the sentences like

All cats can fly.

raised discussions (cf. the previous lesson). The point is that it is very tempting to argue *"No cats can fly"* (which is equivalent to *"All cats can not fly"*), but this is a stronger statement than just the opposite. The opposite would be "Some cats can not fly". Kids naturally tried to revert false statements to absolutely true statements, and this is not always the right way to produce exactly the opposite statement. After several examples on board we noticed that to construct the opposite for these kind of sentences one has to interchange *"All"* with *"Some"* and "*are"* with *"are not"*. (I did not give any explanation why this works, since I did not give any definitions). With that rule in our minds we did the problems in the table. We also observed that in the table several of Nein's possible responses are *true* statements, but only *one* of them is *the opposite*.

Remark for parents. What is this exercise about? It is about the heart and soul of any mathematical proof – about logic. It seems that logic should be a natural ability for everyone, but surprisingly, wrong conclusions or logically wrong reasons surround us in media, politics, ordinary life… (may be, some people do it on purpose ☺). In any case, it is good to know the right way to make logic conclusions (even if you are not going to use that right way – at least then you can track fishy statements of others).

3. The second part was dedicated to binary trees. There was again a question, why is this math-related. Here is a possible answer: these are pre-elements of writing algorithms. We talked a little bit about the 20 question toy. Then we worked with the problem about animals. During the conversation I have had to explain what I really meant in my questions on the handout : if a rabbit is a wild animal, if this is a wild or domestic horse, does the fox sleep during the winter (meaning that they sleep at night during the winter), does the bear sleep during the winter (it seems that in this region they do not!). You see how hard it is to make the precise correct questions! For this reason the vegetable-fruit handout was a little bit harder, since kids were supposed to make such questions themselves. The task was approached with great enthusiasm.
I cite here the questions that I remember to be invented

Lemon vs Apple
-Is it sour? (but some kids argued that apples also can be sour).
-Is it a lemon?
-Does it look like an American football ball?
-Does it start with letter L?

Cherry vs Grape

-Does it have stems ?
-Does it always have stones?
-Is it always not green?

Potato vs Carrot

- Does it look like a stinky (?) nose?
- Does it have skin?
- Is it oval?
- Is it not orange?

Brocolli vs Tomato

- Does it look like a tree?
- Is it eatble when it is green?

Brocoli, Tomato vs Potato, Carrot

-Does it grow above the ground?
- Is this what I do not like?

Apple, Lemon, vs Cherry, Grape

- Is this a berry?

Apple, Lemon, Cherry, Grape vs Brocoli, Tomato, Potato, Carrot

- Is it a vegetable?

4. In 6 pm group we also have had time to play a r game with marbles (and we did it next time in 7 pm group). I hide in a purse 1 or 2 marbles and asked kids to find out in one “yes-no” question, how many marbles there are. This simple exercise entertained the audience a lot. It was a little bit funny to observe satisfaction on the little faces when they opened the purse and there was exactly as many marbles as they predicted.

Then we repeated the same game but with 1,2,3, or 4 marbles and two “yes-no” question. I hide 3 marbles and prepared to answer to two questions, but to my bad luck the first question was “Are there 3 marbles”? So I hide 3 marbles again for the next game, and the next person, who tried to guess with questions “Are there 4 marbles?” , “ Are there 2 marbles?” did not get enough information to find out the number of marbles. Another participant immediately suggested the right strategy: He asked “ Is the number of marbles bigger than 2?” and “Is it 4?”.

Opposites

This is the rule that we are going to observe in the next set of problems:

ALL (…) ARE (…).

is the opposite to

SOME(…) ARE NOT (…).

Clowns Ja and Nein are friends. But whenever clown Ja says something, clown Nein says the ***complete opposite***. For example, if Ja says

- *All cats like dogs!*

Nein would immediately argue:

- *Some cats do not like dogs!*

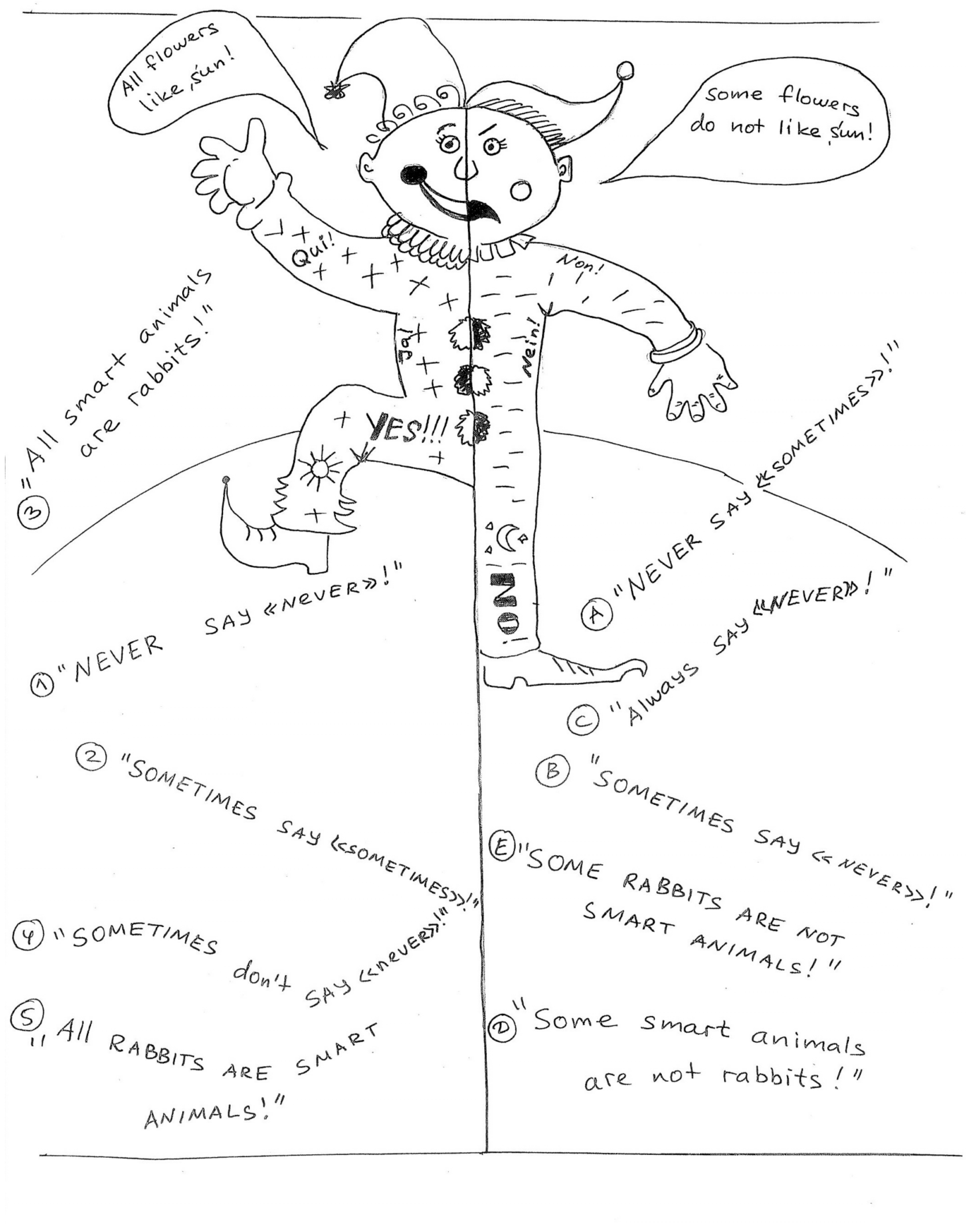
(Note that “ *Some cats like dogs*!” and “*All cats do not like dogs!*” are not the opposites of “All cats like dogs!”)

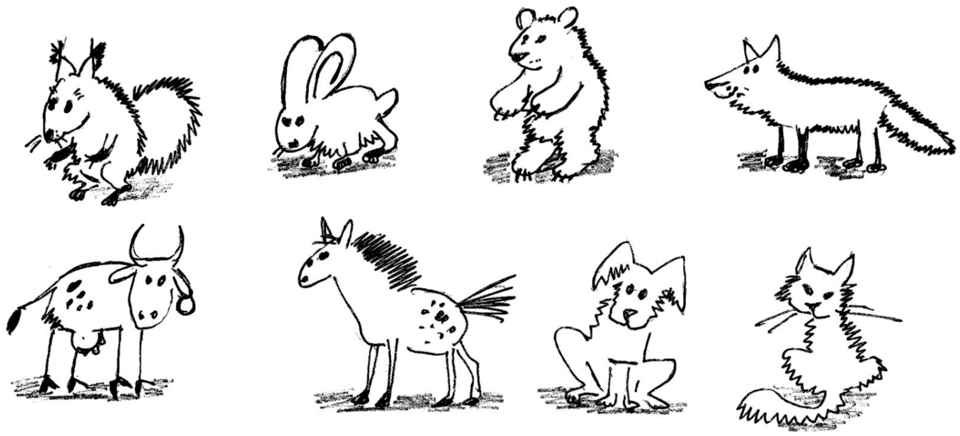
Problem 1.

Ja said	**What did Nein reply? Which statements you think to be true and which you think to be false?**
- *All kids go to school*!	*A. Some kids go to school!* *B. Some kids do not go to school!* *C. All kids do not go to school!*
- *Some books are interesting!*	*B. Some books are not interesting!* *C. All books are interesting!* *D. All books are not interesting!*

Problem 2.

On the picture match the statements of Ja with the answers of Nein.

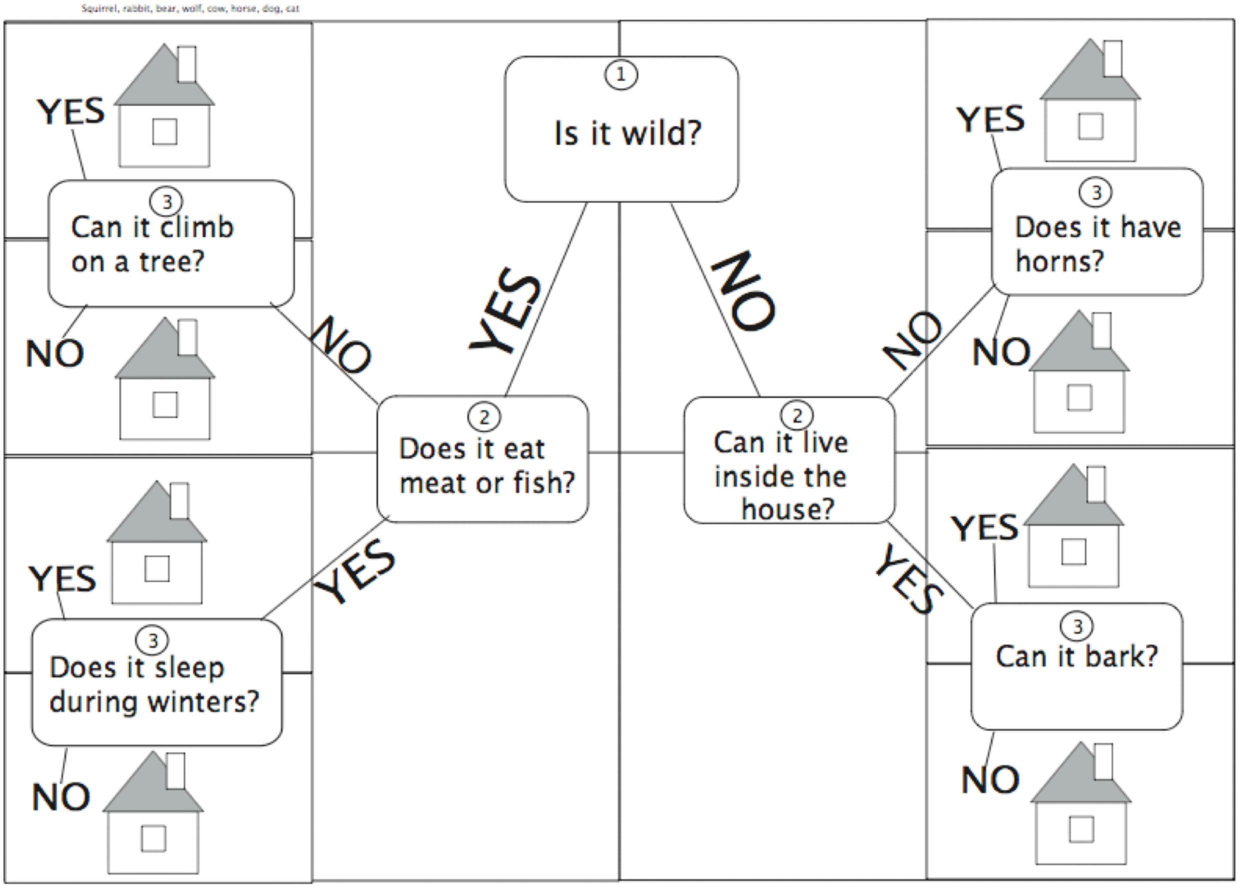




1. Animals

Cut out the pictures of the animals above. Find the right house for each of them: use the questions on the map.

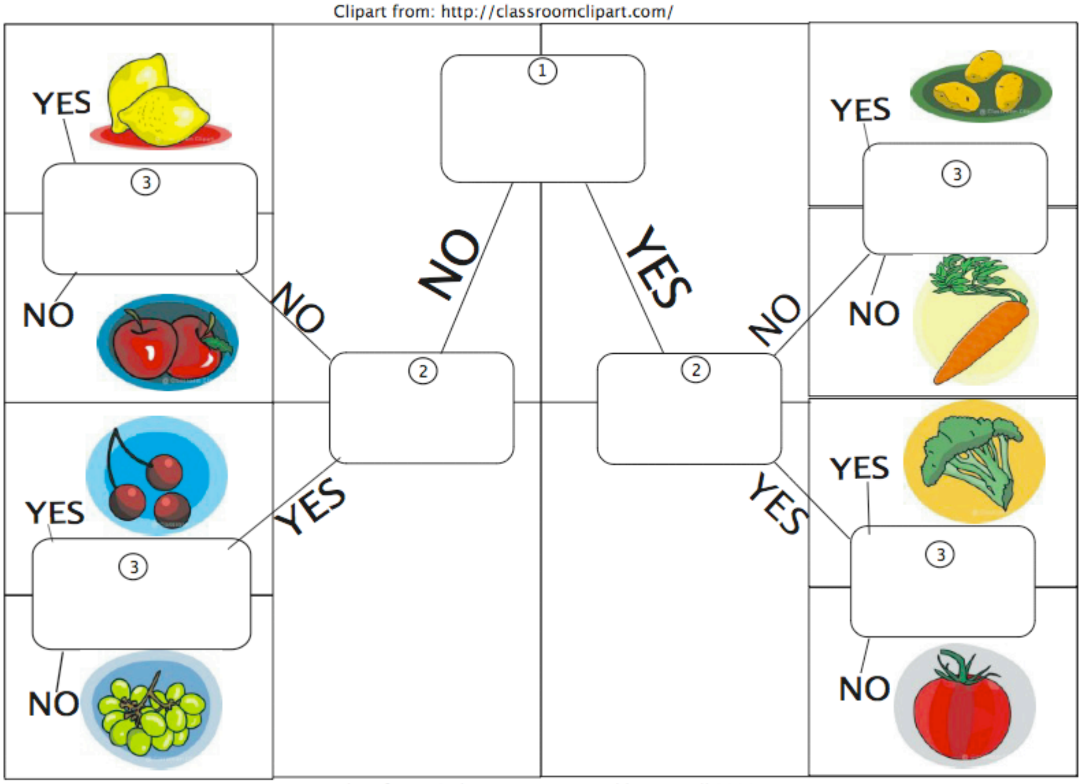
Squirrel, rabbit, bear, wolf, cow, horse, dog, cat



2. Vegetables.

Your friend thought of one of the vegetables and fruits that you can see on the picture. Make a map of questions with yes/no answers, so that in three questions you could guess that vegetable or fruit.

Clipart from: http://classroomclipart.com/



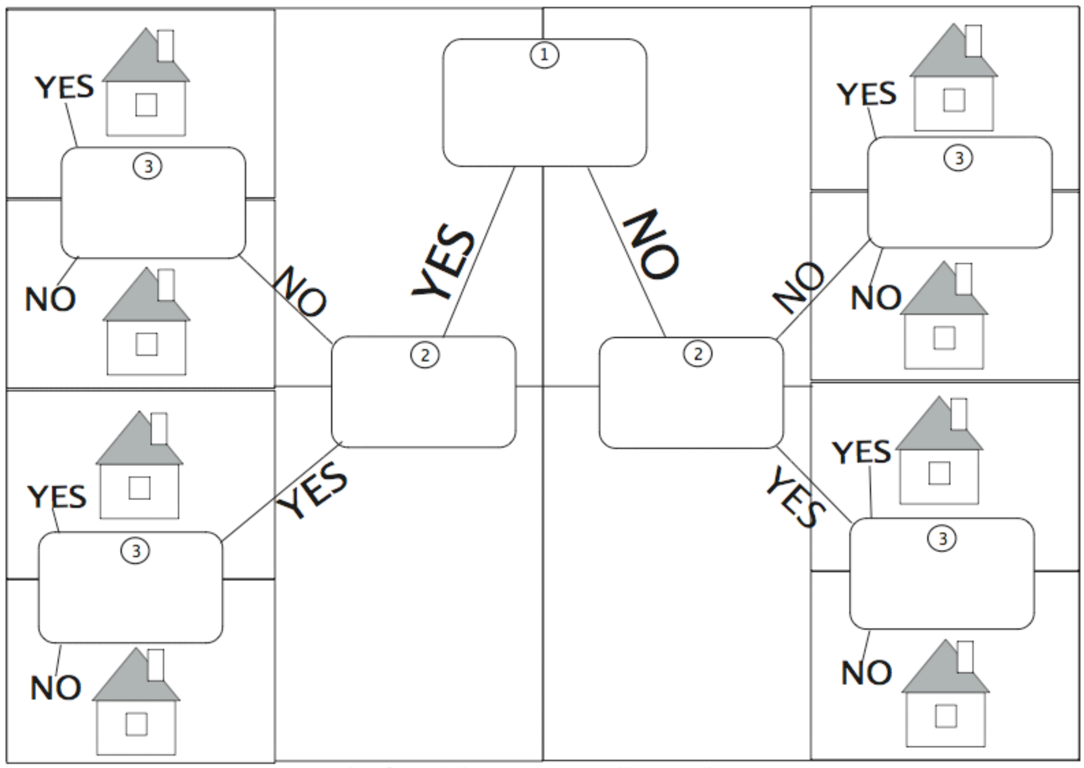
Handout for BMC Elementary. Prepared by NR, Fall 2009

3. Marbles

I have in my pocket few marbles: at least one, but no more than 8. Can you guess in three questions, how many marbles I have?

4. Family.

Write near each house the name of your family member (it can be you, mom, dad, your dog, your brother…) Make a map of "yes-no" questions that would allow to guess your family member in three questions.



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A LITTLE BIT ABOUT 20Q TOY

(for discussion with your parents at home)

You may know about a small toy “20 Q”: you think of some noun (e.g. “the sky”, “a cow”, “happiness”, …) and the little box tries to guess your word by asking 20 question with “yes/no” answers. (Actually, you may also answer “sometimes” and “unknown”). We can count the size of the vocabulary that can be covered (theoretically) by 20 “yes-no” questions.
As we observed in the activities above,

In **1** question we can distinct one word out of the vocabulary of **2** words.

In **2** questions we can distinct one word out of the vocabulary of **2X2=4** words.

In **3** questions we can distinct one word out of the vocabulary of **2X2X2=8** words…

In **20** questions we can distinct one word out of the vocabulary of

2X2X … X2 = 2^20 =1, 048, 576 words,

that is over a million! Look at the average vocabulary count estimates from Wiki below!

In reality there are some adjustments in the algorithm and the toy does not quite follow the binary tree. The human player can make answers that do not match the data of the toy . (For example, how would you answer the question "Is it colorful?", or "Does it bring joy to people?" if you have the word "horse" in your mind?). But anyway, 20 questions collect enough data to guess the word in 70-80% of trials.
The on-line version of 20q can be played at *http://www.20q.net*

Vocabulary Count

Grade 1 Student = 1,000+ words
Normal Person (Graduate) = 5,000 to 6,000+ words
University Professor = 15,000+ words
Spelling Bee Winners = 30,000+ (as claimed by them)
College Dictionary (Abridged) = 50,000 - 70,000
Total Words in English Language = 250,000+ (Growing)
Dictionary (Un-abridged) with derivatives = 450,000+
Shakespeare used 60,000 words

(link: http://wiki.answers.com/Q/How_many_words_in_an_average_vocabulary)