Lesson 12 December 24, 2009 BMC Elementary

Overview.
 The attached handouts on symmetry contain more than we covered in class: in advance it was not clear for me, how far we would go, so I prepared extra problems and varied the questions according to the kids reaction.

1. The logic problem about the dragon went smoothly. The participants not only gave the right answers, but also explained (proved!) them.

2. We started to talk about kaleidoscopes. We have had several kaleidoscopes to share in class. Everyone saw them before, but to my surprise:

a) Not everyone knew the name of the toy.
b) It was hard for some kids to explain what they see: the structure of the picture (symmetry, triangle grid) did not catch their eye: "Just some cool picture". Other kids obviously got the idea that there is symmetry in the picture, but among those not everyone could find the right words to describe. Many used the words "they have equal parts", meaning the symmetry of the picture.
c) After these observations one more was not a surprise: very few of kids actually ever thought, how kaleidoscope works. Some knew that there is "glass inside", but our attempts to estimate the number of mirrors varied from 1 to 18.

   All of above indicated that the topic about mirrors was not as easy as I expected it to be.

2. We did the first activity – constructing the reflections of different figures in one mirror ("When a letter E came to a mirror…). After that we started to talk about two mirrors. Unfortunately, none of instructors was able to find two mirrors at home to illustrate the activity. May be, it would went easier, if we could show real mirrors. So we went just with pictures and draw the reflections of mirrors and the letter J. We observed that the number of reflections varies with the angle between mirrors. We talked about two photos - a girl inside the kaleidoscope and the old lady.
Then on the blackboard I draw the picture of reflections in the room with 3 walls-mirrors, and it was clear that this is as far as we can go. We spent some time drawing kaleidoscope pictures on a grid of triangles, and we checked our understanding of the question on the handout where two pictures can be images from a kaleidoscope, and two can not. Some kids showed very good intuition on that problem, so in general I think our discussion about reflections was fruitful.

3. We also took some time to talk about the notion of symmetry in general. On the blackboard I put several figures made out of K'nex and their "shadows" made on copy machines. One of the figure (a fish) had no symmetries, the rest have had either rotational, or reflection symmetries , or both. I asked, which figure "does not belong", and most of us agreed that it is the one that had no symmetries. In 7 pm group kids gave better explanations, why it does not belong, and even pronounced clearly that it has no symmetries. The figure that have had only rotation symmetry o looked suspicious for us, but we agreed that the fish is even more suspicious. Then I gave the definition of the symmetry – as it is written on the handout below, and I put the K'nex figures over their shadows, and illustrated that some transformations put the figures back exactly on the place over their shadows.

4. Snow flakes - all of different kind with templates from www.KinderArt.com. Since we were a little bit tired already, I did not ask questions about snowflakes that I planed to ask: e.g. what kind of symmetry they have. We just made them.
5. Finally, everyone got a bag with flat polygons (constructor Polydron) and we made different polytopes out of them. I think kids liked it.

Three knights came to the king and told him that they killed a dragon.

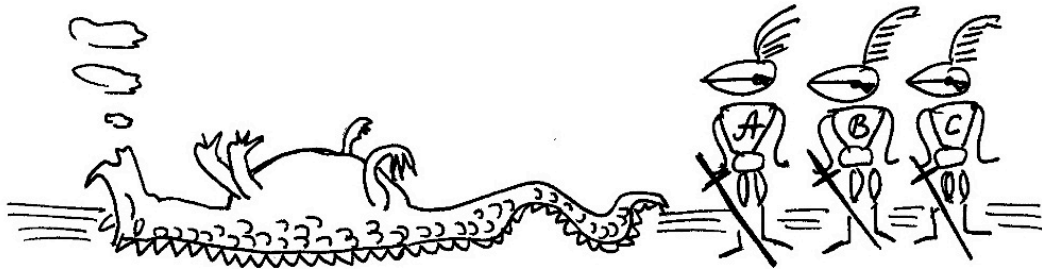Knight A: Knight B killed the dragon!
Knight B: Knight C killed the dragon!
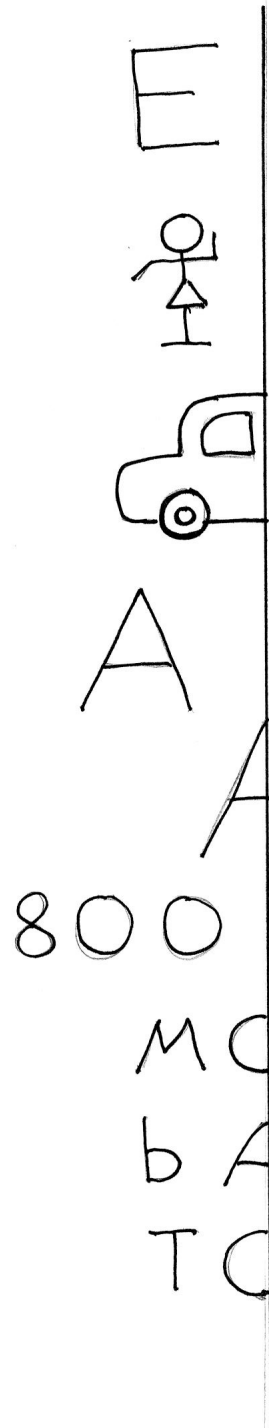Knight C: I killed the dragon.

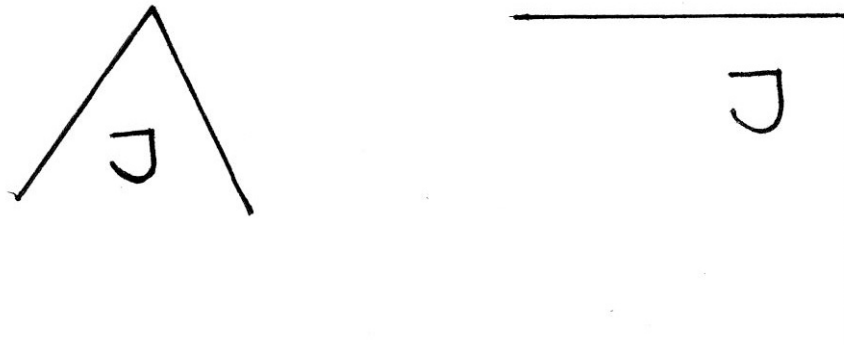Problem 1. They all lied. Who killed the dragon?

Problem 2. Only one of them told the truth. Who killed the dragon?

Problem 3. Exactly two of them told the truth. Who killed the dragon?

When a letter "E" Came to a Mirror, what did it see?. .

When a Mirror Came to a Mirror...

What did a Mirror "see" when it "came to a Mirror"?

What did letter J "see" in the two mirrors?

How many reflections of the letter "J" will be there?

Notice that what you see in two mirrors depends a lot on the angle between two mirrors. Try to experiment with those at home.
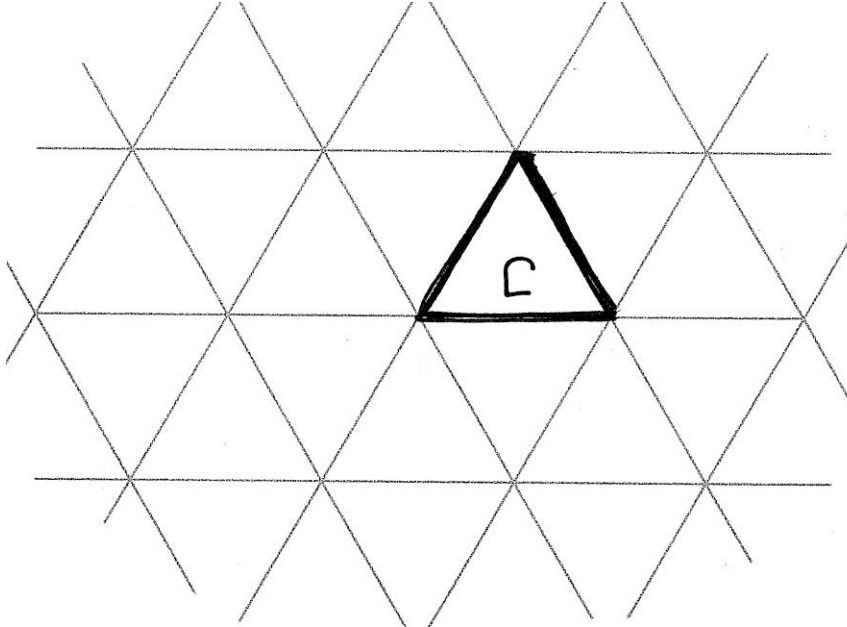
Old Lady

How many mirrors were used for this picture? What can you say about the angle about them? Which lady is real?

Source of the image:http://math.hws.edu/~mitchell/Math110F08/

Three mirrors

1. Draw the images of mirrors (the grid should help you to do this).
2. Draw the images of letter "J" as they appear in the mirrors and in the reflections of the mirrors
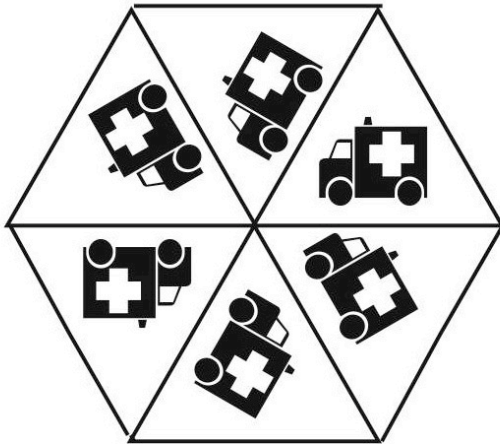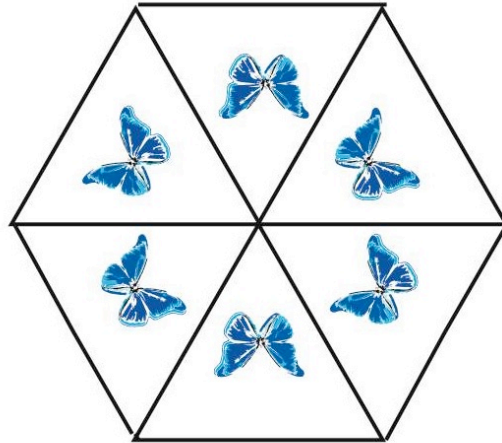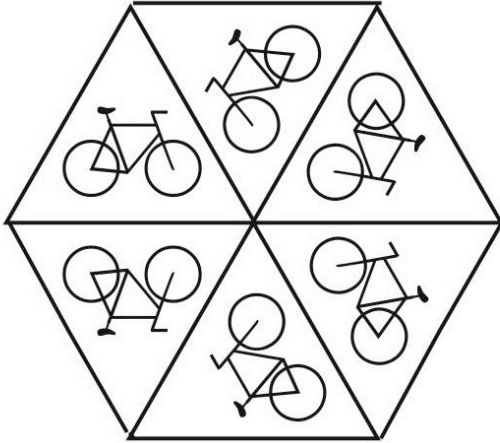

This is how you would feel if you would get inside the Kaleidoscope. Actually, you can experience that feeling – in Exploratorium.

Source of image: bnl.gov

Kaleidoscope

Which of the following pictures cannot appear in kaleidoscope?
Using the grid of triangles, draw your own kaleidoscope picture.

Symmetry of an object means that you make a transformation of a space (or a plane) that contains your object, and the object comes back to itself under this transformation. We will talk today only about *reflections* and *rotations.*

Any reflection on the plane is defined by a mirror.

Any rotation of the plane is defined by a center and an angle.

For the next set of pictures answer to the following questions:

1. Does it have reflection symmetry? If yes, draw the mirrors.
2. Does it have rotation symmetry?
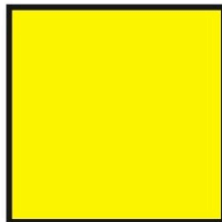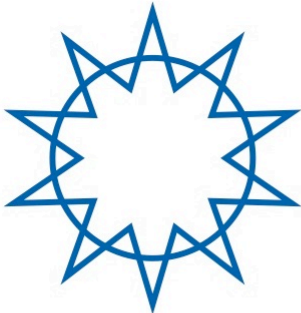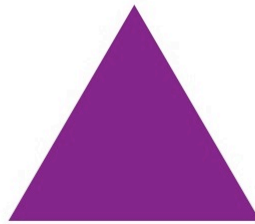
Four mirrors. What would you see inside a Kaleidoscope with 4 mirrors? For example, here letter J is inside the room where all 4 walls are mirrors. What does it see?
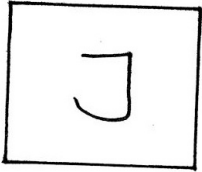

Paper Snowflakes
1. Make a paper snowflake. For templates one can go, for example to www.KinderArt.com
2. What kind of symmetries do we have in paper snowflakes?

Observe that we produce beautiful snowflakes by symmetric repetition of a small region (sometimes this region is called *the fundamental domain*). This is true for any symmetric pattern. Going back to the pictures on the previous page, find the fundamental domains for each of them.

POLYTOPES
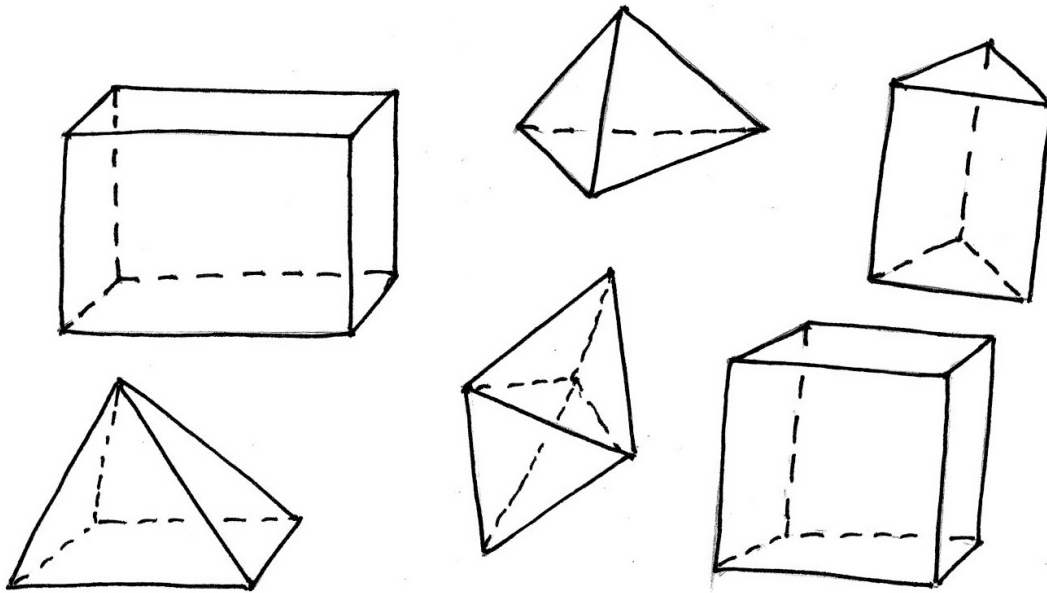
Out of the pieces that you got you should be able to construct a polytope. Three-dimensional objects also can have rotational and reflection symmetries.

In 3D rotation is defined by an axes and by an angle.

Reflection is defined by a mirror –plane.

1. Find the rotational and reflection symmetries of your polytope.
2. Count the following numbers

F= Number of Faces =

E=Number of Edges=

V= Number of Vertices=

Now find :

F+V-E=

(This is a theorem in mathematics, that for any polytope
F-E+V=2)