Dynamical Systems

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Introduction

What is $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}}}$? How about $2^{2^{2^{2^{2^{\cdots}}}}}$? For what a does $a^{a^{a^{a^{a^{\cdots}}}}}$ make sense? What is its value?

Problems

1. What happens if we repeat the rule $x \to x^2 - 1$? Does our result depend on our seed value x_0?

2. How about the doubling map

$$D : x \to \begin{cases} 2x & \text{if } x \leq \frac{1}{2} \\ 2x - 1 & \text{if } x > \frac{1}{2} \end{cases}$$

if we pick the seed x_0 to be between 0 and 1? Are there any attractive fixed points? Are there any attractive cycles?

3. How about the tent map

$$T : x \to \begin{cases} 2x & \text{if } x \leq \frac{1}{2} \\ 2\,(1 - x) & \text{if } x > \frac{1}{2} \end{cases}$$

if we pick the seed x_0 to be between 0 and 1? Are there any attractive fixed points? Are there any attractive cycles?

4. Suppose we change the multiplier from 2 to 3 in our tent map. For what seed values between 0 and 1 will the iterates stay between 0 and 1?

5. Use an iterative map to answer the questions in the introduction.

Computer Exercises

6. What happens if you program the maps in problems **2** and **3** into a computer? On a computer, for what seed values between 0 and 1 will the iterates from problem **4** stay between 0 and 1? (Hint: think binary.)

7. How does the behavior of the iterates of the logistic map

$$f_c : x \to cx\,(1 - x)$$

change as c varies from 1 to 4?