Coins in Twoland

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In Twoland, the only money is coins with value 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, and so on.

There is also a law that requires all purchases be made with exact change. Only the banks are allowed to make change.

Oneville

In Oneville, there is an additional law: purchases must be made with at most one copy of each coin. So for instance, paying 5 must be 4 + 1 (or 1 + 4, but we'll consider that the same). It would be illegal in Oneville to hand over 2 + 2 + 1 in payment.

1. How many ways are there to pay 1023?

2. How many ways are there to pay 1024?

3. How many ways are there to pay 1000?

Where do we live?

4. How many ways are there to write 349 in our system? Of course we can't write $2_{14}9$ with a 14 in the tens place - why not? What are the laws?

Twoville

Twoville, the capital of Twoland, turns out to be a much more interesting place. Their law requires that payments be made with at most two copies of each coin. So now you can pay 5 with 4+1 or 2+2+1, so there are two ways to do it.

5. How many ways are there to pay 1023?

6. How many ways are there to pay 1024?

7. How many ways are there to pay 1000?

8. How many different methods can we find for solving that last problem?