# Berkeley Math Circle
# Monthly Contest 6
# Due March 3, 2009

**Instructions**

This contest consists of 5 problems, some of which are easier than the others. Every problem is worth 7 points. Please, write solution to every problem on a separate sheet of paper, and on top of each sheet include your name, grade and school, as well as the problem number and the contest number. Thus, the header on each sheet should look something like:

Solution to Problem 3 of BMC Monthly Contest 5
by Bart Simpson
in grade 5
from Springfield Middle School, Springfield

If you submit more than one sheet for a specific problem, please, staple the sheets together to avoid getting them confused with someone else's solution. Please, do NOT staple together solutions to DIFFERENT problems, as they will be graded separately.

Carefully justify your answers to avoid losing points. Include all relevant explanations in words and all intermediate calculations. Answers without justification will receive no credit. However, good reasoning with minor calculational errors may receive a lot of points. Thus, submit solutions to as many problems as you can since partial credits will be awarded for sufficient progress on any particular problem.

Remember that you are NOT ALLOWED to consult or talk to anyone else about the problems, whether in person, on the phone, via e-mail, or other means of communication. You can consult any book that you wish. For more on the contest rules, please, check the BMC website at http://mathcircle.berkeley.edu.

Enjoy solving these problems and good luck!

**Problems**

1. Let $p$, $q$, and $r$ be distinct primes. Prove that $p + q + r + pqr$ is composite.

2. The sequence
$$5,\ 9,\ 49,\ 2209, \ldots$$
is defined by $a_1 = 5$ and $a_n = a_1 a_2 \cdots a_{n-1} + 4$ for $n > 1$. Prove that $a_n$ is a perfect square for $n \geq 2$.

3. The integers from 1 to 13 are arranged around several rings such that every number appears once and every ring contains at least one two-digit number. Prove that there exist three one-digit numbers adjacent to one another on one ring.

4. Let $ABC$ be a triangle with $\angle ABC = 90^\circ$. Points $D$ and $E$ on $AC$ and $BC$ respectively satisfy $BD \perp AC$ and $DE \perp BC$. The circumcircle of $\triangle CDE$ intersects $AE$ at two points, $E$ and $F$. Prove that $BF \perp AE$.

5. Let $a_1,\ a_2, \ldots,\ a_n$ be distinct integers. Prove that there do not exist two nonconstant integer-coefficient polynomials $p$ and $q$ such that
$$(x - a_1)(x - a_2) \cdots (x - a_n) - 1 = p(x)q(x)$$
for all $x$.