Bay Area Mathematical Olympiad
and Mathematical Circles

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Berkeley Math Circle
Monthly Contest 3
Due November 23, 2003

1. Find all integer solutions to $xy = 2003(x + y)$

2. Show that for each $n \geq 17$ one can cut a square into n smaller squares.

3. In each cell of $n \times n$ table there is an arrow pointing in one of eight principal directions (i.e. one of the following: $\rightarrow, \leftarrow, \uparrow, \downarrow, \nearrow, \searrow, \nwarrow, \swarrow$), in such a way that the arrows in any two adjacent cells form an angle of no more than 45 degrees, and the arrows in any two cells adjacent diagonally (i.e. sharing a vertex) form an angle of no more than 90 degrees. We start at some cell and follow the arrows (e.g. if the arrow in our cell is $\uparrow$ we move up by one, if it is $\searrow$ we move diagonally down and to the right, etc.). Prove that we will eventually escape from the table (i.e. will reach a cell on the boundary of the table where the arrow will point "out of the table").

4. Let $\mathbf{R}^+$ be the set of all positive real numbers. Find all functions f : $\mathbf{R}^+ \to \mathbf{R}^+$ such that

$$f(x)f(yf(x)) = f(x + y)$$

for all $x, y \in \mathbf{R}^+$.

5. A circle is tangent to the continuations of sides CA and CB of the triangle ABC, and is also tangent to the side AB at point P. Prove that the radius of the circle tangent to AP, CP and the circle circumscribed around ABC is equal to the radius of the circle inscribed in ABC.