Berkeley Math Circle Monthly Contest #8
Due April 28

1. In triangle ABC, let H, D, M be the feet of the altitude, angle bisector, and median from A. Prove that D is between H and M.

2. Prove that, for $n \geq 6$ an equilateral triangle can be dissected into n equilateral triangles.

3. Solve

$$\begin{aligned} (x+y)^3 &= z \\ (y+z)^3 &= x \\ (z+x)^3 &= y \end{aligned}$$

   for real x, y, z.

4. Find all $f : \mathbb{N} \to \mathbb{N}$ such that

$$f(n+1) > f(f(n)).$$

5. Prove that there exists a line l in the plane of triangle ABC such that the area of the intersection of triangle ABC and its reflection over l has more than $2/3$ the area of triangle ABC.