Berkeley Math Circle Monthly Contest # 7

Due March 31, 2002

1. Find all ordered pairs of integers (a, b) such that

$$a^3 - b^3$$

is prime.

2. One day, while walking along, Alya stumbles into an infinite chessboard with the primes written on it. 2 is in the lower right-hand corner, and every prime appears only once on the board. Alya starts on the 2 with no apples. Upon entering a square with the number p, she either picks up or drops p apples. Can she end up on the 2 with no apples?

3. Find all functions f such that

$$f(x-1) + f(x+1) = 6x + f(x).$$

4. During a break, n Berkeley math professors sit in a circle around a graduate student to play a game. The student walks clockwise around the professors and hands out math problems according to the following rule. He selects one professor and gives him a problem, then he skips the next professor and gives a problem to the next, then he skips two and gives a problem to the next, then he skips three, and so on. Determine the values of n for which eventually all of the professors will be too distracted with problems to give the student homework. (i.e. everyone has a problem.)

5. Show that for any integer $n \geq 6$ there exists a convex hexagon which can be dissected into exactly n congruent triangles.