Berkeley Math Circle Monthly Contest # 1
Due September 30, 2001

1. Show that in any finite gathering of people, there are at least two people who know the sme number of people. (Knowing is symmetrical; if I know you, you know me. Also, we consider only "knowing" other people - one does not "know" oneself.)

2. The numbers $1, 2, \ldots, n$ are written on the board. Every minute, I erase two numbers a and b and replace them with $a + b + ab$. Clearly, after $n - 1$ minutes there is only one number left on the board. Does the result depend on the order? For which n can I make the resulting number be odd?

3. Let $a, b, c \geq 0$. Prove
$$(a + b)(b + c)(c + a) \geq 8abc.$$

4. Consider $\triangle ABC$. Let G be the intersection (other than A) of the circle with diameter AB and the circle with diameter AC. Let D be the intersection of the altitude from C with AG. Prove that the radius of the circumcircle of $\triangle ABD$ is equal to the radius of the circumcircle of $\triangle ACD$.

5. How many ways can you tile a $3 \times n$ rectangle with 2×1 dominoes? State the answer explicitly in terms of n.