

Berkeley Math Circle Monthly Contest #8
Due April 28

1. In triangle ABC , let H, D, M be the feet of the altitude, angle bisector, and median from A . Prove that D is between H and M .
2. Prove that, for $n \geq 6$ an equilateral triangle can be dissected into n equilateral triangles.
3. Solve

$$\begin{aligned}(x + y)^3 &= z \\(y + z)^3 &= x \\(z + x)^3 &= y\end{aligned}$$

for real x, y, z .

4. Find all $f : \mathbb{N} \rightarrow \mathbb{N}$ such that

$$f(n + 1) > f(f(n)).$$

5. Prove that there exists a line l in the plane of triangle ABC such that the area of the intersection of triangle ABC and its reflection over l has more than $2/3$ the area of triangle ABC .