Berkeley Math Circle Monthly Contest #6
Due March 3

1. Find all integer solutions to
$$x^2 - 3y^2 = 17.$$

2. The point P lies inside an equilateral triangle and its distances to the three vertices are $3, 4, 5$. Find the area of the triangle.

3. Suppose that $n = a^2 + b^2 + c^2$ where a, b, c are positive integers. Prove that for any positive integer k, n^{2k} can be written $A^2 + B^2 + C^2$, where A, B, C are positive integers.

4. You are placing letters from an alphabet of n letters around a circle by the following rules:

   (i) No two adjascent letters are the same

   (ii) For any two different letters a and b it is possible to draw a line through the circle such that all of the a's are on one side, and all of the b's are on the other side.

   Find the maximum number of letters that you can place around the circle.

5. Prove that the boundary of the intersection of n circles consists of at most $2n - 2$ circular arcs.