Berkeley Math Circle Monthly Contest #1
Due September 24, 2000

1. Two sets of points in the coordinate plane are given: [(-1,1), (-1, 2), . . . , (-1, 2000)} and {(1, 1), (1, 2), . . . , (1, 2000)}. 2000 line segments are drawn connecting these points so that each point in the first set is connected to exactly one point in the second set, and vice versa. Find, with proof, the sum of the y-intercepts of the segments.
2. Find all functions f from the set **R** of real numbers to itself such that $f(xy + 1) = x f(y) + 2$ for all $x, y \in \mathbf{R}$.
3. Let ABC be a triangle and D, E, F, G, H, I, J points on sides BC, CA, AB, BC, CA, AB, BC, respectively, such that EF and HI are parallel to BC; IJ and FG are parallel to CA; and DE, GH are parallel to AB. Prove that $D = J$.
4. There are 30 members of the math faculty at UC Berkeley. The evil Gastropod places a red or green stamp on each member's forehead, then places them all in separate rooms. Each day, the Gastropod convenes all the faculty in one room and asks any member who has conclusively determined that his/her stamp is red to raise his/her hand. No other communication between them is allowed. As it turns out, everyone has a red stamp (but since each can only see the other's stamps, none of them realize this at first. The Gastropod remarks on the first day: "At least one of you has a red stamp." Given that all Berkeley faculty members are perfect reasoners, and each knows all the others are perfect reasoners, prove that everyone will eventually determine that his/her stamp is red.
5. Let $a_1, a_2, \ldots, a_{2000}$ be real numbers in the interval [0, 1]. Find the maximum possible value of

$$\sum_{1 \le i < j \le 2000} (j - i)|a_j - a_i|.$$

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, problem number, and the division in which you are participating (beginner or advanced). Please go to **http://mathcircle.berkleey.edu** for more information about the contest, or email questions to gastropod@hotmail.com or dudzik@yahoo.com. © Berkeley Math Circle