Berkeley Math Circle Monthly Contest #4
Due January 7, 2001

1. Two players play a game with pennies, which are circles of radius 1, on an $m \times n$ rectangular table. Each player takes turns putting a penny on the table so that it touches no other penny. The first player who is unable to do so loses. The table starts with no pennies on it. Assuming that there is an infinite supply of pennies, for what values of m and n does the first player have a winning strategy?

2. Suppose that k and n are integers with $0 \leq k \leq n-3$. Prove that
$$\binom{n}{k}, \binom{n}{k+1}, \binom{n}{k+2}, \binom{n}{k+3}$$
cannot form an arithmetic progression.

3. A point M and a circle k are given in the plane. If $ABCD$ is an arbitrary square inscribed in k, prove that the sum $MA^4 + MB^4 + MC^4 + MD^4$ is independent of the positioning of the square.

4. Find all pairs of integers x, y such that $2x^2 - 6xy + 3y^2 = -1$.

5. Suppose that $S_1, S_2, S_3, \ldots$ are sets of integers such that no integer is contained in more than one S_n; every S_n has exactly two elements; and the sum of the elements of S_n is n. Prove that there exist infinitely many values of n with the following property: one of the elements of S_n is greater than $13n/7$.

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, problem number, and the division in which you are participating (beginner or advanced). Please go to **http://mathcircle.berkeley.edu** for more information about the contest, or email questions to gastropodc@hotmail.com or dudzik1@yahoo.com.