Berkeley Math Circle Monthly Contest #3
Due November 19, 2000

1. I have three opaque jars filled with coins. One contains only nickels; one contains only dimes; one contains some nickels and some dimes. Also, the three jars are labeled "Nickels," "Dimes," and "Nickels and Dimes," but it is known that all of the jars are labeled incorrectly. You are allowed to choose one jar and look at one coin randomly drawn from it. How can you determine which jar is which?

2. An $n \times n$ matrix of integers is called "golden" if, for every row and every column, their union contains all of the numbers $1, 2, 3, \ldots, 2n-1$. Find all golden matrices (of all sizes).

3. An equilateral triangle in the coordinate plane has vertices $(a, b), (c, d), (e, f)$. Prove that a, b, c, d, e, f cannot all be integers.

4. Let f, g be two functions from the set $\mathbf{R}$ of real numbers to itself, such that $f(x) < g(x)$ for all $x \in \mathbf{R}$. Prove that there exists an infinite subset $S \subseteq \mathbf{R}$ such that $f(x) < g(y)$ for all $x, y \in S$.

5. A permutation of the numbers $1, 2, \ldots, n$ is called "bad" if it contains a subsequence of 10 numbers in decreasing order, and "good" otherwise. For example, for $n = 15$,

$$15, 13, 1, 12, 7, 11, 9, 8, 10, 6, 5, 4, 3, 2, 14$$

is a bad permutation, because it contains the subsequence

$$15, 13, 12, 11, 10, 6, 5, 4, 3, 2.$$

Prove that, for each n, the number of good permutations is at most 81^n.

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, problem number, and the division in which you are participating (beginner or advanced). Please go to **http://mathcircle.berkeley.edu** for more information about the contest, or email questions to gastropodc@hotmail.com or dudzik1@yahoo.com.