Berkeley Math Circle Monthly Contest #2
Due October 22, 2000

1. Given $n+1$ distinct integers from the set $\{1, 2, \ldots, 2n\}$ $(n \geq 1)$, prove that some two of them are relatively prime.

2. Three grasshoppers are on a straight line. Every second one grasshopper jumps. It jumps across one (but not across two) of the other grasshoppers. Prove that after 1999 seconds the grasshoppers cannot be in the initial position.

3. Do there exist three different prime numbers such that the sum of any two of them is a square?

4. Let $S = \{1, 2, \ldots, n\}$, and let T be the set consisting of all nonempty subsets of S. The function $f : T \to S$ is "garish" if there do not exist sets $A, B \in T$ such that A is a proper subset of B and $f(A) = f(B)$. Determine, with proof, how many garish functions exist.

5. Let $ABCD$ be a quadrilateral, and let O be the intersection of AC and BD. Quadrilateral $A'B'C'D'$ is obtained by rotating ABCD around O by some angle. Let A_1, B_1, C_1, D_1 be the intersection points of the lines $A'B'$ and AB, $B'C'$ and BC, $C'D'$ and CD, and $D'A'$ and DA, respectively. Prove that quadrilateral $A_1B_1C_1D_1$ is cyclic if and only if AC is perpendicular to BD.

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, problem number, and the division in which you are participating (beginner or advanced). Please go to **http://mathcircle.berkeley.edu** for more information about the contest.