# BAMO 2000: PRACTICE PROBLEMS II

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*Note:* You have 4 hours to solve as many problems as you can from the following list of 5 problems. Each solution should be written clearly and in detail on a separate sheet of paper. Each problem is worth 7 points. Partial credit will be awarded for partial solutions.

**Problem 1.** Prove that there are no integers $x$ and $y$ satisfying the equation $x^2 - 3y^2 = 17$.

**Problem 2.** $\triangle ABC$ is inscribed in a circle $k$ with diameter $BD$. Let $H$ be its orthocenter.

(a) Prove that point $D$ is symmetric to $H$ with respect to the midpoint $F$ of side $AC$.

(b) If $AC = DH$, find the measure of $\angle ABC$.

**Problem 3.** Let $x_1,\ x_2, ...,\ x_n$ be non-negative numbers whose sum is $1/2$. Prove that

$$\frac{1-x_1}{1+x_1} \cdot \frac{1-x_2}{1+x_2} \cdots \frac{1-x_n}{1+x_n} \geq \frac{1}{3}.$$

For what $x_i$'s is equality achieved?

**Problem 4.** Let $a_1,\ a_2, ...., a_n, ...$ be a sequence of positive numbers such that $a_n^2 \leq a_n - a_{n+1}$ for all $n = 1, 2, ...$ Prove that $a_n \leq 1/n$ for all $n$.

**Problem 5.** Let $a_1, a_2, ..., a_{mn+1}$ be a sequence of distinct numbers. Prove that we can find either an increasing subsequence of length greater than $m$ or a decreasing subsequence of length greater than $n$.