Berkeley Math Circle April 16, 2000

1. Find nonnegative integers such that $79x + 179y = 7319$.

2. Prove that if a, b, c are integers then $ax + by = c$ has a solution in integers x and y if and only if c is divisible by the greatest common divisor of a and b.

3. Suppose that a and b are relatively prime, i.e., that $\gcd(a, b) = 1$. Show that the largest n that is not of the form $ax + by = n$ for positive integers x and y is $n = ab$. Find the largest non-representable integer when x and y are merely required to be nonnegative.

4. Show that the number of steps required by Euclid's algorithm when applied to two integers is at most 5 times the number of decimal digits in the largest of the integers.

5. Find the "worst case" integers for Euclid's algorithm. Find the worst case integers when the least-remainder form of Euclid's algorithm is used.

6. Find a "simultaneous" recursion for pairs (x_n, h_n) where x_n $x_n, x_n + 1, h_n$ are the three sides of a Pythagorean triangle.

7. Describe all integers that are simultaneously square and triangular numbers.

8. Show that the determinant of the product of 2×2 matrices is the product of their determinants.

9. Find the continued fraction of $179/79$. What is the relationship between this and Euclid's algorithm?

10. Find a integer solution to solution $x^2 - 23y^2 = 1$.

11. Find a integer solution to solution $x^2 - 43y^2 = 1$.

12. If $x^2 - y^2 d < \sqrt{d}$ show that

$$\left| \frac{x}{y} - \sqrt{d} \right| < \frac{1}{2y^2}.$$