BERKELEY MATH CIRCLE MONTHLY CONTEST #7, DUE APRIL 23, 2000

(1) Equilateral triangle ABC is inscribed in a circle. Let D be the midpoint of AB, and let E be the midpoint of AC. The ray $\overrightarrow{DE}$ meets the circle at P. Prove that $DE^2 = DP \cdot PE$.

(2) Prove that there are infinitely many terms in the arithmetic progression

$$8, 21, 34, 47, \ldots$$

which consist entirely of 9's.

(3) Let $p(x)$ be a polynomial of degree exactly 3, with real coefficients. For each real number a, let $q_a(x)$ be the unique polynomial of degree 2 or less such that $p(x) - q_a(x)$ is divisible by $(x - a)^3$. Prove that for $a \neq b$, the graphs of $q_a(x)$ and $q_b(x)$ do not intersect. Can you find a generalization to polynomials $p(x)$ of odd degree?

(4) Find the minimum of the function $f(x, y)$

$$f(x, y) = \sqrt{(x+1)^2 + (2y+1)^2} + \sqrt{(2x+1)^2 + (3y+1)^2} + \sqrt{(3x-4)^2 + (5y-6)^2},$$

defined for all real $x, y > 0$.

(5) Let O be the center of a circle k. Points A, B, C, D, E, F on k are such that triangles OAB, OCD, OEF are equilateral. Let L, M, N be the midpoints of BC, DE, FA, respectively. Prove that triangle LMN also is equilateral.

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, and problem number. Remember that these problems are not to be discussed with anyone until after the due date. Please go to
`http://www.math.berkeley.edu/~stankova/MathCircle/Joyce/index2.html`
for more information about the contest.

Date: March 26, 2000.