BERKELEY MATH CIRCLE MONTHLY CONTEST #3, DUE 12/12/99

1. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(x) + f(y) \neq 0$ and
$$\frac{f(x) - f(x-y)}{f(x) + f(x+y)} + \frac{f(x) - f(x+y)}{f(x) + f(x-y)} = 0$$
for all $x, y \in \mathbb{R}$.
2. Prove that for $n \geq 1$, the last $n + 2$ digits of 11^{10^n} are $6000\ldots0001$, with n zeros between the 6 and the final 1.
3. Let A_1, B_1, C_1 be points in the interior of sides BC, CA, and AB, respectively, of equilateral triangle ABC. Prove that if the radii of the inscribed circles of ΔC_1AB_1, ΔB_1CA_1, ΔA_1BC_1, $\Delta A_1B_1C_1$ are equal, then A_1, B_1, C_1 are the midpoints of the sides of ΔABC on which they lie.
4. Let $0 < \alpha < 1$. Prove that there exists a real number x, $0 < x < 1$, such that $\alpha^n < \{nx\}$ for every positive integer n. (Here $\{nx\} = nx - \lfloor nx \rfloor$ is the fractional part of nx.)
5. Pentagon $ABCDE$ is cyclic, i.e., inscribed in a circle. Diagonals AC and BD meet at P, and diagonals AD and CE meet at Q. Triangles ABP, AEQ, CDP, and APQ have equal areas. Prove that the pentagon is regular; i.e., all its sides are equal, and all its angles are equal.

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, and problem number. Remember that these problems are not to be discussed with anyone until after the due date. Please go to `http://www.math.berkeley.edu/~stankova/MathCircle/Joyce/index2.html` for more information about the contest.

Date: November 7, 1999.