BERKELEY MATH CIRCLE MONTHLY CONTEST #2, DUE 11/7/99

1. A 6×6 square is covered by nonoverlapping dominos (2×1 rectangles, placed horizontally or vertically). Prove that there must be a horizontal line or a vertical line that passes through the interior of the big square, but which does not cut the interior of any domino. (Hint: it's possible to prove this without checking cases. Another hint: can a horizontal line or vertical line cut exactly one domino?)
2. The set of positive integers is partitioned into finitely many subsets. Show that some subset S has the following property: for every positive integer n, S contains infinitely many multiples of n.
3. The Cannibal Club of California (CCC) had 30 members yesterday morning — but that was before their festive annual dinner! After the dinner, it turned out that among any six members of the club, there was a pair one of whom ate the other. Prove that at least six members of the CCC are now nested inside one another (#1 inside #2 inside #3 etc. until #6).
4. Let E be an ellipse that is not a circle. For which $n \geq 3$ is it possible to inscribe a regular n-gon in E? (For each $n \geq 3$, either show how to construct such an n-gon, or show that none exists.)
5. Prove that
$$\tan\left(\frac{3\pi}{11}\right) + 4\sin\left(\frac{2\pi}{11}\right) = \sqrt{11}.$$

Please write solutions to different problems on separate pages. At the top of each page, write your name, school, city, contest number, and problem number. Remember that these problems are not to be discussed with anyone until after the due date. Please go to
`http://www.math.berkeley.edu/~stankova/MathCircle/Joyce/index2.html`
for more information about the contest.

Date: October 10, 1999.