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Diophantine equations. [1]

Homework Problems

1. Solve the following Diophantine equations:
(a)$161x + 7y = 1$;
(b)$4x + 13y = 1$;
(c)$3x + 5y = 41$;
(d)$xy = x + y$;
(e)$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$.

2. Show that the equations below do not have integer solutions:
(a)$x^4 - y^4 = 10$;
(b)$x^2 + y^2 = 7z^2$;
(c)$x^2 - 3y^2 = 2$;
(d)$x^2 - 3y^2 = 7$;
(e)$x^2 - 2y^2 = 3$

3. Write recurrent formulae for all integer solutions of the following equations:
(a)$x^2 - 2y^2 = 2$;
(b)$x^2 - 3y^2 = 1$;
(c)$x^2 - 3y^2 = 6$.

4. Show that if p is prime then the equation $x^2 - py^2 = 1$ has infinitely many solutions.

(a) Show that one can find an integer b such that $x^2 - py^2 = b$ has infinitely many solutions.

(b) Let (x_1, y_1) and (x_2, y_2) be two solutions of the equation from (a), x_1 and x_2 have the same remainder when divided by b and y_1 and y_2 have the same remainder when divided by b. Prove that $\frac{x_1+y_1\sqrt{p}}{x_2+y_2\sqrt{p}}$ belongs to $Z[\sqrt{p}]$ and has norm 1.

(c) Using (b) obtain infinitely many solutions for $x^2 - py^2 = 1$.

5. Obtain a formula for all rational solutions of $x^2 + y^2 = 1$. For this consider an inversion on the plane with center at the point $(0, 1)$ and radius 2. Show that the inversion maps the x-axis to the unit circle with center at the origin. Prove that a rational point on x-axis moves to a point with rational coordinates on the circle.