BERKELEY MATH CIRCLE 2000-2001

Practice Exam I for BAMO 2001

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1. A "crocodile" is a new chess-piece whose move consists of a jump of one cell horizontally or vertically, and then a jump of N cells in the perpendicular direction. (When $N = 2$, a "crocodile" is the usual knight in chess.) For what N can the "crocodile" move from any cell to any other cell on an *infinite* chess board?
2. Point M inside a convex quadrilateral $ABCD$ is such that the area of $\triangle ABM$, $\triangle BCM$, $\triangle CDM$ and $\triangle DAM$ are equal. Is it true that $ABCD$ is a parallelogram, and point M - the intersection of its diagonals?
3. There are 20 kids in some elementary school in Russia. Each pair of kids have a common grandfather. Prove that there is a grandfather with at least 14 grandchildren in the school.
4. A brother and a sister cut a triangular cake as follows: the brother determines a point on the cake, and the sister cuts the cake through that point and chooses one of the two pieces for herself. Each of them wants to get as much cake as possible. Which point should the brother choose? How much of the cake will each of them get in this case? (Note: the triangular shape of the cake doesn't need to be equilateral.)
5. Baron Munhausen told George Cantor that he can write in a sequence all natural numbers without 1's in their decimal representation so that only a finite number of them are bigger than their number (place) in the sequence. Isn't Baron Munhausen lying?