

Trees

1. Sixty hungry piranhas are released in a pond with no food, so they start eating each other. A piranha is considered *satisfied* if it ate at least three other piranhas, regardless of whether it got eaten afterwards. Find the largest possible number of piranhas that could be satisfied.

2. King Arthur had five sons. Among his male descendants, 100 had exactly three sons each and the remaining ones had no children. Is it possible to uniquely determine how many of king's male descendants were childless?

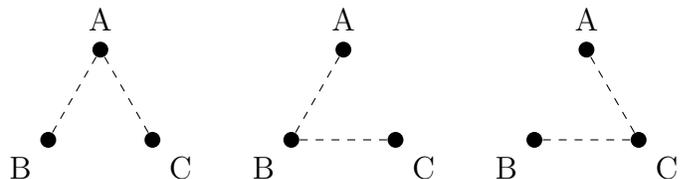
3. A volleyball net has the form of a rectangular lattice made out 50×200 squares whose sides are formed by strings of length one. Determine the maximum number of these unit strings that can be cut before the net falls apart into more than one piece?

4. The country Effiland has 100 cities. The government wants to design a very efficient airline network that would allow to travel by air from any city to any other (perhaps with several changes of plane) but only in one possible way. Show that in any such a network there would always be at least one city which is connected by a direct flight to only one other city.

5. What is the minimum number of flights that Effiland's air network can have? What is the maximum number of flights?

6. Show that a tourist wishing to visit all 100 cities in Effiland can do this using no more than:
 - (a) 198 flights; (b) 196 flights.

7. A group of n servers needs to be connected into a network using $n-1$ cables so that each server could communicate with every other. For example, for three servers A, B, C , this can be done in three different ways:



In how many ways can such a network be made out of (a) 4; (b) 5 servers?