BMC Beginners II: Modular Arithmetic

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Definition: For positive integer m, a and b are congruent modulo m if a and b have the same remainder when divided by m.

$$a \equiv b \bmod m$$

Use the notation for modular arithmetic when solving the following problems.

1. If we have a group of 38 people and we want to make groups of 3, how many people are not in a group?

2. Today is a Wednesday. What day of the week is it 30 days from now?

3. Your neighbor starts building their fence on Monday morning. They work only on weekdays (Mon-Fri), and it takes them 32 days to build the fence. On what day does your neighbor finish building the fence?

4. On 9pm on Tuesday, PG&E had a planned power outage for 100 hours. What day and time does the electricity come back?

5. Suppose Sally is on a snowboarding expedition on the moon. She wants to break the world record and complete a 3150°rotation. If she starts facing due west and turns counter-clockwise, what direction should she be facing at the end?

6. A gaggle of turkeys live on Farmer John's farm. Farmer John sends his 3 kids out to count the number of turkeys. The 3 kids, Alice, Bob, and Charles, come up with a plan. Alice will count by 3's, Bob will count by 5's, and Charles will count by 19's. After counting the turkeys, they report back to Farmer John.

   Alice says: "I counted the turkeys by 3's and there were 2 left over."

   Bob says: "I counted the turkeys by 5's and there were 3 left over."

   Charles says: "I counted the turkeys by 19's and there were 7 left over."

   What is the minimum number of turkeys that Farmer John can have?

Properties: Suppose $a_1 \equiv b_1 \bmod m$ and $a_2 \equiv b_2 \bmod m$. Then, we have the following.

- $a_1 + a_2 \equiv b_1 + b_2 \bmod m$

- $a_1 \cdot a_2 \equiv b_1 \cdot b_2 \bmod m$

Use these properties when solving the following problems.

7. Find a divisibility rule for 8. Make sure to explain why it works.

8. Testing for divisibility for 24 using 4 and 6 doesn't work. Can you find a way that does work?

9. Let n be a natural number. n^2 has a remainder of 4 when divided by 5 and n^3 has a remainder of 2 when divided by 5. What is the remainder of n when divided by 5?