

BERKELEY MATH CIRCLE

The Math of Chemistry:

Moles & Molecules I

Patricio Angulo

The Periodic Table

Elements are organized into the Periodic Table of Elements. They are organized into columns by their similarities in chemical properties:

Periodic Table of the Elements

1 IA 11A																	18 VIII A 8A
1 H Hydrogen 1.008																	2 He Helium 4.003
3 Li Lithium 6.941	4 Be Beryllium 9.012											5 B Boron 10.811	6 C Carbon 12.011	7 N Nitrogen 14.007	8 O Oxygen 15.999	9 F Fluorine 18.998	10 Ne Neon 20.180
11 Na Sodium 22.990	12 Mg Magnesium 24.305	3 IIIB 3B	4 IVB 4B	5 VB 5B	6 VIB 6B	7 VIIB 7B	8 VIII 8	9 VIII 8	10 VIII 8	11 IB 1B	12 IIB 2B	13 Al Aluminum 26.982	14 Si Silicon 28.086	15 P Phosphorus 30.974	16 S Sulfur 32.066	17 Cl Chlorine 35.453	18 Ar Argon 39.948
19 K Potassium 39.098	20 Ca Calcium 40.078	21 Sc Scandium 44.956	22 Ti Titanium 47.88	23 V Vanadium 50.942	24 Cr Chromium 51.996	25 Mn Manganese 54.938	26 Fe Iron 55.933	27 Co Cobalt 58.933	28 Ni Nickel 58.693	29 Cu Copper 63.546	30 Zn Zinc 65.39	31 Ga Gallium 69.732	32 Ge Germanium 72.61	33 As Arsenic 74.922	34 Se Selenium 78.09	35 Br Bromine 79.904	36 Kr Krypton 84.80
37 Rb Rubidium 84.468	38 Sr Strontium 87.62	39 Y Yttrium 88.906	40 Zr Zirconium 91.224	41 Nb Niobium 92.906	42 Mo Molybdenum 95.94	43 Tc Technetium 98.907	44 Ru Ruthenium 101.07	45 Rh Rhodium 102.906	46 Pd Palladium 106.42	47 Ag Silver 107.868	48 Cd Cadmium 112.411	49 In Indium 114.818	50 Sn Tin 118.71	51 Sb Antimony 121.760	52 Te Tellurium 127.6	53 I Iodine 126.904	54 Xe Xenon 131.29
55 Cs Cesium 132.905	56 Ba Barium 137.327	57-71 Lanthanide Series	72 Hf Hafnium 178.49	73 Ta Tantalum 180.948	74 W Tungsten 183.85	75 Re Rhenium 186.207	76 Os Osmium 190.23	77 Ir Iridium 192.22	78 Pt Platinum 195.08	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.383	82 Pb Lead 207.2	83 Bi Bismuth 208.980	84 Po Polonium [208.982]	85 At Astatine 209.987	86 Rn Radon 222.018
87 Fr Francium 223.020	88 Ra Radium 226.025	89-103 Actinide Series	104 Rf Rutherfordium [261]	105 Db Dubnium [262]	106 Sg Seaborgium [266]	107 Bh Bohrium [264]	108 Hs Hassium [269]	109 Mt Meitnerium [268]	110 Ds Darmstadtium [269]	111 Rg Roentgenium [272]	112 Cn Copernicium [277]	113 Uut Ununtrium unknown	114 F1 Flerovium [289]	115 Uup Ununpentium unknown	116 Lv Livermorium [298]	117 Uus Ununseptium unknown	118 Uuo Ununoctium unknown
		57 La Lanthanum 138.906	58 Ce Cerium 140.115	59 Pr Praseodymium 140.908	60 Nd Neodymium 144.24	61 Pm Promethium 144.913	62 Sm Samarium 150.36	63 Eu Europium 151.966	64 Gd Gadolinium 157.25	65 Tb Terbium 158.925	66 Dy Dysprosium 162.50	67 Ho Holmium 164.930	68 Er Erbium 167.26	69 Tm Thulium 168.934	70 Yb Ytterbium 173.04	71 Lu Lutetium 174.967	
		89 Ac Actinium 227.028	90 Th Thorium 232.038	91 Pa Protactinium 231.036	92 U Uranium 238.029	93 Np Neptunium 237.048	94 Pu Plutonium 244.064	95 Am Americium 243.061	96 Cm Curium 247.070	97 Bk Berkelium 247.070	98 Cf Californium 251.080	99 Es Einsteinium [254]	100 Fm Fermium 257.095	101 Md Mendelevium 258.1	102 No Nobelium 259.101	103 Lr Lawrencium [262]	

Alkali Metal	Alkaline Earth	Transition Metal	Semimetal	Nonmetal	Basic Metal	Halogen	Noble Gas	Lanthanide	Actinide
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FROM LAST TIME:

*For each element, we can directly relate the amount of protons, electrons and neutrons that exist.
But first, we need to learn some terms!*

Symbol of Element

1 or 2 letter abbreviation for each element

Mass Number

- *Not always a whole number (more on this later!)*
- *#protons + #neutrons*

Atomic Number

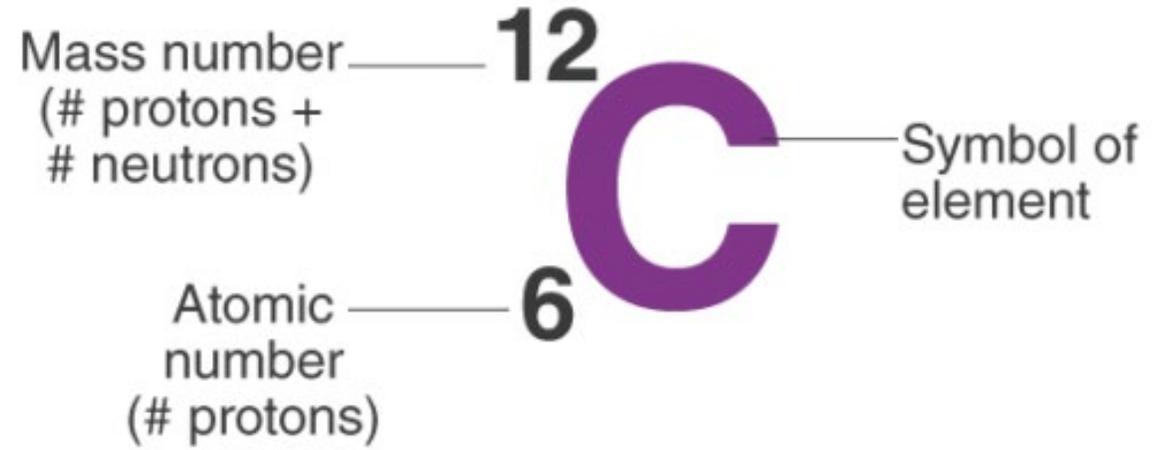
#protons (defines the element!)

Thus,

of protons = atomic number (defines the element!)

of electrons = # of protons (if neutral)

of neutrons = Mass Number - Atomic number



When we think of atoms, we can't "see" them, but we can measure them. We call this a **microscopic** view. But what about a **macroscopic** view? What things can we see, and then also measure? And how do we measure them?

For example:



If we measure our examples above, how do we compare them? By weight? By mass?

Since all of these elements exist as a collection of whole atoms, the only way to compare or measure them on **equal standing** is by measuring and counting **how many** of them exist. Their mass may be different, but the amount of them may be the same. In essence, we are counting how many particles of each element or molecule exist. Any guesses how many atoms in each of these?



Since we are dealing with very, very small objects, we need a way to count them in an easily digestible manner. Just like 1 dozen = 12 and 1 kilometer = 1000m, we need small, digestible values to represent INSANELY huge amounts!

We therefore welcome the concept of **Avogadro's Number** (NOT Avocado!)



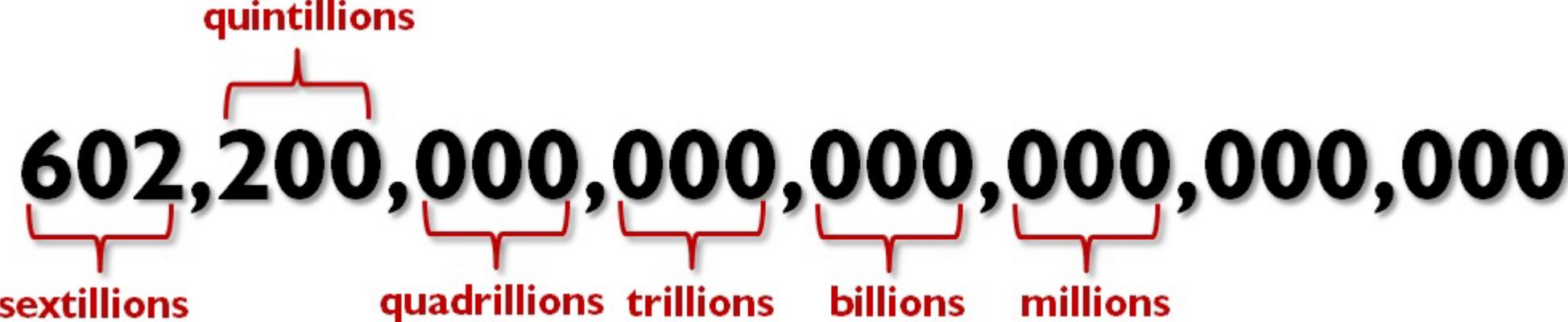
Avogadro \neq **Avocado**

Avogadro's number is **6.022×10^{23}** . That's a HUGE amount!

How huge? Well

602,200,000,000,000,000,000,000,000,000,000

602.2 Sextillion!



What then, does this HUGE number represent?

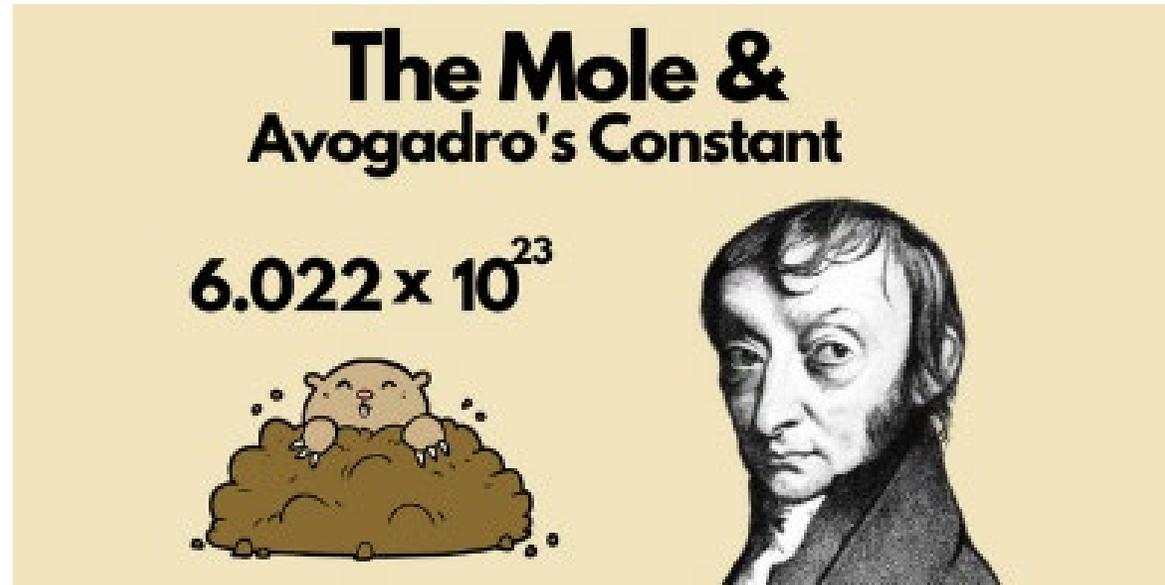
$6.022 \times 10^{23} = 1$ **mole** of things (atoms, particles, molecules)

Again, just like:

1 dozen = 12

1 kilo = 1000

1 mole = 6.022×10^{23}



So, we've learned two new concepts:

Avogadro's Number AND the Mole

How does all relate back to our microscopic and macroscopic view of things? From before:

Since all of these elements exist as a collection of whole atoms, the only way to compare or measure them on equal standing is by measuring and counting how many of them exist.

So:

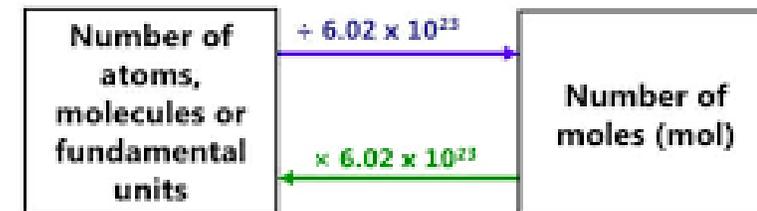
1 mole of atoms = Avogadro's number of atoms

1 mole of atoms = 6.022×10^{23} atoms

A mole is a unit of quantity.

A mole is 6.02×10^{23} things.

6.02×10^{23} is known as Avogadro's constant (N_A)



Again, from before:

Their mass may be different, but the amount of them may be the same. In essence we, are counting how many particles of each element or molecule exist.

We will relate all of this to mass soon, but first, let's play with scientific notation, powers of ten, and numbers of things!

Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2 moles of carbon. How many atoms of carbon do I have?

2) I have 0.5 moles of salt (NaCl). How many molecules of salt do I have?

3) I have 5 moles of water (H₂O). How many molecules of water do I have?

Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2 moles of carbon. How many atoms of carbon do I have?

$$\frac{2 \text{ moles Carbon}}{1} \times \frac{6 \times 10^{23} \text{ atoms of Carbon}}{1 \text{ mole of Carbon}} = 2 \times (6 \times 10^{23}) = 12 \times 10^{23} = \mathbf{1.2 \times 10^{24} \text{ atoms of carbon}}$$

2) I have 0.5 moles of salt (NaCl). How many molecules of salt do I have?

$$\frac{0.5 \text{ moles NaCl}}{1} \times \frac{6 \times 10^{23} \text{ molecules of NaCl}}{1 \text{ mole of NaCl}} = 1/2 \times (6 \times 10^{23}) = \mathbf{3 \times 10^{23} \text{ molecules of NaCl}}$$

3) I have 5 moles of water (H₂O). How many molecules of water do I have?

$$\frac{5 \text{ moles H}_2\text{O}}{1} \times \frac{6 \times 10^{23} \text{ molecules of H}_2\text{O}}{1 \text{ mole of H}_2\text{O}} = 5 \times (6 \times 10^{23}) = 30 \times 10^{23} = \mathbf{3 \times 10^{24} \text{ molecules of H}_2\text{O}}$$

More Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2.4×10^{28} atoms of Calcium (Ca). How many moles of Calcium do I have?

2) I have 3×10^{20} molecules of carbon dioxide (CO₂). How many moles of CO₂ do I have?

3) I have 1.8×10^{25} particles of helium gas (He). How many moles of helium do I have?

More Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. You DO NOT need a calculator ;)

1) I have 2.4×10^{28} atoms of Calcium (Ca). How many moles of Calcium do I have?

$$\frac{2.4 \times 10^{28} \text{ atoms of Calcium}}{1} \times \frac{1 \text{ mole of Calcium}}{6 \times 10^{23} \text{ atoms of Calcium}} = \frac{2.4 \times 10^{28}}{6 \times 10^{23}} = \frac{24 \times 10^{27}}{6 \times 10^{23}} = 4 \times 10^4 = \mathbf{40,000 \text{ moles Ca}}$$

2) I have 3×10^{20} molecules of carbon dioxide (CO₂). How many moles of CO₂ do I have?

$$\frac{3 \times 10^{20} \text{ molecules CO}_2}{1} \times \frac{1 \text{ mole of CO}_2}{6 \times 10^{23} \text{ atoms of CO}_2} = \frac{3 \times 10^{20}}{6 \times 10^{23}} = \frac{1 \times 10^{20}}{2 \times 10^{23}} = 0.5 \times 10^{-3} = 5 \times 10^{-4} = \mathbf{0.0005 \text{ moles CO}_2}$$

3) I have 1.8×10^{25} particles of helium gas (He). How many moles of helium do I have?

$$\frac{1.8 \times 10^{25} \text{ particles He}}{1} \times \frac{1 \text{ mole of He}}{6 \times 10^{23} \text{ particles He}} = \frac{1.8 \times 10^{25}}{6 \times 10^{23}} = \frac{18 \times 10^{25}}{6 \times 10^{23}} = 3 \times 10^2 = \mathbf{300 \text{ moles He}}$$

BUT, how does this relate to what we see and measure in day-to-day life! From before, we had the objects below.
Can we say?

“I have 6.022×10^{23} atoms of carbon in my pencil”

“Please pass me 3.01×10^{23} particles of salt”

“May you fill my balloon with 1.04×10^{24} atoms of Helium?”

Of course not! **That’s ridiculous!**

We tend to measure things by their mass, and then relate them to how many of them exist (atoms, molecules, moles, etc.). So, we need to a way to do so

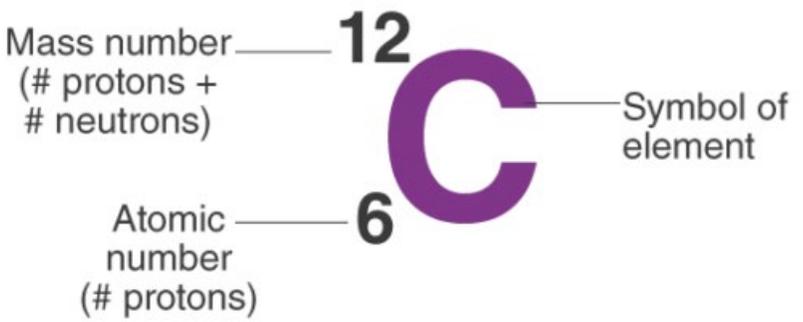


Our good friend, **The Periodic Table of Elements** is here to help us again! Without getting into all of the background, scientists have discovered that an Avogadro's amount of any element or atom = 1 mole of that element or atom, and is ALSO equal to that element or atom's atomic mass when measured in **grams**.

Periodic Table of the Elements

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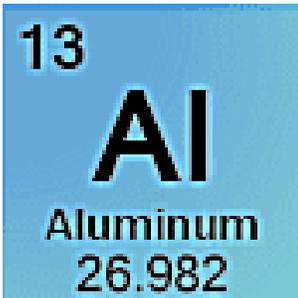


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For Example:

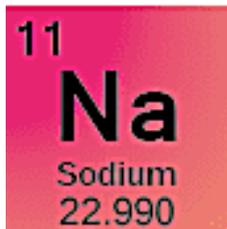


6.022×10^{23} atoms of Lithium
1 mole of Lithium
6.941 grams of Lithium

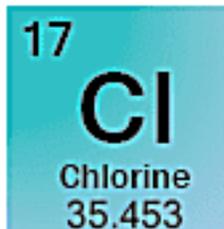


6.022×10^{23} atoms of Aluminum
1 mole of Aluminum
26.982 grams of Aluminum

It's also ADDITIVE! Table Salt = Sodium Chloride



+



=

6.022×10^{23} molecules of NaCl

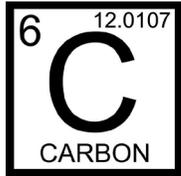
1 mole of NaCl

$22.990 + 35.453 = 58.443$ grams of NaCl

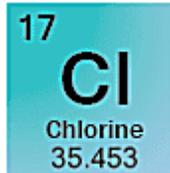
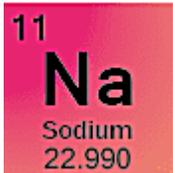
Even More Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

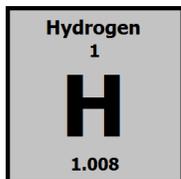
1) I have 2 moles of carbon. How many grams of carbon do I have?



2) I have 0.5 moles of salt (NaCl). How many grams of salt do I have?



3) I have 5 moles of water (H₂O). How many grams of water do I have?

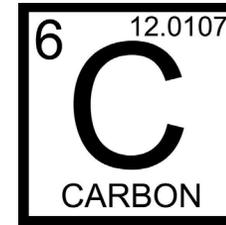


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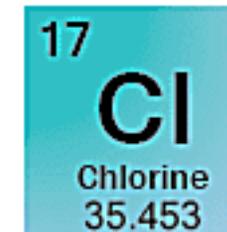
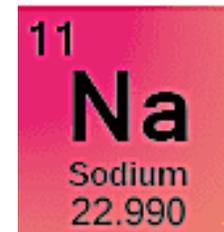
1) I have 2 moles of carbon. How many grams of carbon do I have?

$$\frac{2 \text{ moles of Carbon}}{1} \times \frac{12 \text{ grams Carbon}}{1 \text{ mole of Carbon}} = 24 \text{ grams of Carbon}$$



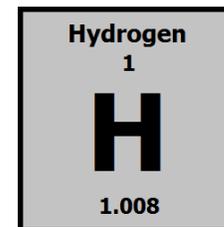
2) I have 0.5 moles of salt (NaCl). How many grams of salt do I have?

$$\frac{0.5 \text{ moles of NaCl}}{1} \times \frac{58.5 \text{ grams NaCl}}{1 \text{ mole of NaCl}} = 29.3 \text{ grams of NaCl}$$



3) I have 5 moles of water (H₂O). How many grams of water do I have?

$$\frac{5 \text{ moles of H}_2\text{O}}{1} \times \frac{18 \text{ grams H}_2\text{O}}{1 \text{ mole of H}_2\text{O}} = 90 \text{ grams of H}_2\text{O}$$



More than Even More Math Fun!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

- 1) I have 80.2 grams of Calcium (Ca). How many moles of Calcium do I have? How many atoms of Calcium do I have?

- 2) I have 132 grams carbon dioxide (CO₂). How many moles of CO₂ do I have? How many molecules of CO₂ do I have?
BONUS Question: How many atoms of Oxygen do I have?

- 3) I have 30 grams of helium gas (He). How many moles of helium do I have? How many particles of He do I have?

The MOST Math Fun EVER!

To simplify, let's use $6 \times 10^{23} = 1$ mole of things for Avogadro's Number. Also, round off mass numbers to their first decimal space. For example, Nitrogen's mass number is listed as 14.007, so use 14.0.

- 1) Pick your favorite Element from the Periodic Table. Write down it's element abbreviation and mass number.
- 2) Square your age (use whole numbers only).
- 3) Your age squared is how many grams of your favorite element you have.
- 4) Calculate how many moles of your element you have.
- 5) Calculate how many atoms of your element you have.

Bonus Problem

Someone in class asked about what some of these items may look like in real life. The most familiar example is water (H_2O). If you poured a glass of water, how many water molecules are in the glass, i.e., how many molecules of water are you drinking?

Let's assume 1 cup of water:

1 cup = 8 fluid ounces = 237 mL (you can look up this conversion factor)

We need water's density to match up with we learned last week, which is 1g/mL.

So, 1 standard cup of water of contains 237 grams of water. We therefore calculate:

You can do similar thought problems with salt, or any other pure (or mostly pure) substance in your house.

What Have We Learned?

We relate quantities in Chemistry but how many PHYSICAL items exist. This is why knowing Avogadro's number and the concept of a mole is so important! If we are trying to calculate specifically what is happening, we MUST know how many things are interacting with how many other things, and we do so via moles. Therefore, the relationship below is KEY to understanding chemical interactions:

of Particles \leftrightarrow Moles \leftrightarrow Grams

Let's Expand This!

Let's use our NEW understanding to understand something we probably ALL have done: the famous at-home volcano experiment!

Does anyone know what is happening in that experiment?



Let's Expand This!

The basics are that we mix vinegar with baking soda and BOOM! But what's happening on a chemical level?

Vinegar contains Acetic Acid (CH_3COOH). This is found in liquid form.

Baking Soda contains sodium bicarbonate (NaHCO_3). This is found in solid form.

So,



And then



Carbon Dioxide is a gas which then escapes into the air. This is what causes the “lava” movement in the experiment!

Relating back to Earlier . . .



One can relate how much acetic acid and sodium carbonate will react (assuming a full reaction) by using our previous relationship and expanding on it

Previously we had . . .

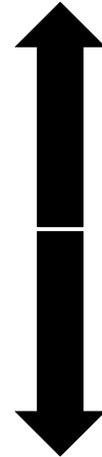
of Particles \leftrightarrow Moles \leftrightarrow Grams

And we can use this for BOTH the acetic acid and sodium carbonate.

BUT, how do we relate them to each other?

The Mole Bridge!

of Particles \leftrightarrow **Moles** \leftrightarrow **Grams**



of Particles \leftrightarrow **Moles** \leftrightarrow **Grams**

The Mole Bridge!

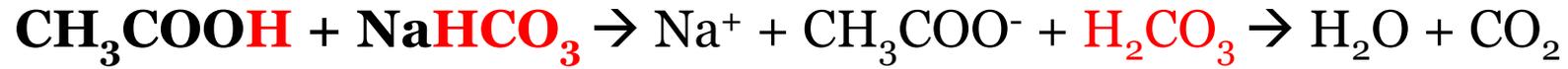
The mole bridge is part of a concept in Chemistry called Stoichiometry.

Stoichiometry = the relationship between the relative quantities of substances taking part in a reaction

Back to volcanoes then

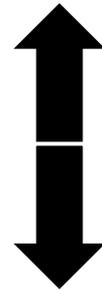


Volcano Problem



If I used 100 grams of Acetic Acid, how many grams of Sodium Bicarbonate will I need to measure out in order to react all of the 100 grams of Acetic Acid?

of Particles ↔ Moles ↔ Grams



of Particles ↔ Moles ↔ Grams

Volcano Problem

CH_3COOH → 60g/1 mole

NaHCO_3 → 84g/1 mole

100 grams of CH_3COOH \times $1/60 = 5/3$ moles CH_3COOH →
 $5/3$ moles NaHCO_3 \times 84g/mole = **140g NaHCO_3**

of Particles ↔ Moles ↔ Grams



of Particles ↔ Moles ↔ Grams