1   Points and Distances

1. Draw a number line, and plot and label these numbers: 0, 1, 6, -2

2. Plot the following points:

(A) (3,1)

(B) (2,5)

(C) (-1,1)

(D) (2,-4)

(E) (-3,-3)

(F) (0,4)

(G) (-2,0)

(H) (0,0)

3. Find the **distance** between:

(i) Point B and point C.

(ii) Point F and point G.

(iii) Point C and point E.

(iv) Point A and point G.

4. Plot two points which have a distance of 10 between them. Plot two points which have a distance of $\sqrt{2}$ between them.

2   Area

5. On a new set of axes, plot the following points: $A = (0,0)$, $B = (0,3)$, $C = (4,0)$, $D = (3,5)$, $E = (-2,0)$. Then, find the **areas** of the triangles:

(i) ABC

(ii) ABD

(iii) BCE

(iv) BCD

6. Draw a new set of axes, and have your neighbor draw a triangle on your sheet. What are the coordinates of the vertices of the triangle? Can you find the area of your triangle?

7. Find the area of the quadrilateral $ABCD$, where $A = (-1, -1)$, $B = (2, 1)$, $C = (0, 3)$, $D = (-2, 2)$.

3   Lines and Slopes

8. Draw a new set of axes. For each pair of points, plot them on the axes, find the coordinates of the **midpoint** of the points, and the **slope** of the line segment between the points.

  (i) $(0, 0)$ and $(2, 4)$

 (ii) $(-1, -2)$ and $(1, 0)$

(iii) $(0, -3)$ and $(3, -2)$

(iv) $(-5, 0)$ and $(-2, 3)$

9. List the coordinates of four points on each line, and plot them on a grid.

  (i) The line through $(2, 0)$ with slope 1.

 (ii) The line through $(4, -1)$ with slope $1/2$.

(iii) The line through $(-3, 2)$ with slope -1.

(iv) The line through $(-1, -3)$ with slope 2.

10. The **y-intercept** of a line is the number b such that $(0, b)$ is on the line. Find the y intercept of all the lines in part 9.

11. Find the equation of each line in number 9. Then, for each line, find the value of y when $x = 100$.

12. Find the equation of the line passing through the two points:

  (i) $(-1, 1)$ and $(0, 2)$.

 (ii) $(0, 1)$ and $(2, 5)$

(iii) $(-2, -1)$ and $(4, -3)$

13. Find the equation of the line parallel to the given line that passes through the point $(0, 5)$.

  (i) The line through $(2, 0)$ with slope 1.

(ii) The line through $(4, -1)$ with slope $1/2$.

14. Find lines perpendicular to the given lines that pass through the point $(0,0)$:

  (i) $y = x$

 (ii) $y = 2x$

(iii) $y = 3x$

(iv) $y = x/2$

15. Find the perpendicular bisector of the line segment between $(0,0)$ and $(2,4)$.

Challenge Problems

  (i) The **circumcenter** of a triangle is the intersection of the perpendicular bisectors of its sides. Find the orthocenter of the triangle with vertices at $(2, -3)$, $(-2, 6)$, $(-2, -8)$.

 (ii) The **orthocenter** of a triangle is the intersection of its altitudes. Find the orthocenter of the triangle from (i).

(iii) Find all possibilities for the fourth vertex of a parallelogram which has three vertices at $(-2, -1)$, $(3, 1)$, $(-1, 5)$.

(iv) Find the equations of the two lines through $(-1, 0)$ which make a 45 degree angle with the line $y = 2x + 2$.

4   Circles

16. List twelve points on the circle centered at $(1, 2)$ with radius 5, whose coordinates are integers.

17. What is the equation of the circle centered at $(1, 2)$ with radius 5?

18. Write the equation of the following circles:

  (i) The circle centered at $(-1, 1)$ with radius 2.

 (ii) The circle centered at $(0, -3)$ with radius 3.

(iii) The circle centered at $(-2, -2)$, passing through $(-1, 3)$.

(iv) The circle centered at $(4, -1)$, passing through $(2, 1)$.

 (v) The circle centered at $(0, 0)$ with radius 1.

19. Find the center and radius of the circles described by the following equations:

(i) $(x - 1)^2 + (y - 1)^2 = 16$

(ii) $(x + 2)^2 + (y - 7)^2 = 49$

(iii) $x^2 + (y + 1)^2 = 4$

(iv) $(x - 100)^2 + (y - 100)^2 = 100$

Challenge Problems

(i) There are two lines which pass through the point $P =$, and which intersect the circle with center $O =$ and radius r exactly once. Find the slopes of those lines.

(ii) A line has slope m, and intersects the circle with radius 1 centered at $(0, 0)$ in two points: $(-1, 0)$, and (x, y). Find x and y in terms of m.

(iii) The **circumcircle** of a triangle is a circle which passes through each of its vertices. Find the circumcircle for the triangle with vertices at $(-3, -2)$, $(1, 2)$, and $(-5, 8)$.

5   Transformations

Translations

20. Let T denote translation five units to the right. Find the result of transforming the following points, and plot both the point and its translation.

(A) $(0, 0)$

(B) $(-2, 3)$

(C) $(-5, -1)$

(D) $(1, -2)$

21. Let S denote translation 3 units upward. Find and plot $S(A)$, $S(B)$, $S(C)$, and $S(D)$, on the same graph as before.

22. Let R denote the transformation T followed by the transformation S. Find and plot $R(A)$, $R(B)$, $R(C)$, and $R(D)$. Is this the same as transformation S followed by the transformation T?

23. On a new graph, plot the point $A = (2, 2)$, and the following points:

(i) $T(A)$, where T is translation by the vector $(3, 1)$.

(ii) $S(A)$, where S is translation by the vector $(-1, -4)$.

(iii) $R(A)$, where R is translation by the vector $(-7, 2)$.

(iv) $Q(A)$, where Q is translation by the vector $(1, -2)$.

24. If a translation T sends a point $A = (-1, -3)$ to the point $T(A) = (3, -1)$, by what vector is T a translation? Where does T send $(0, 2)$?

25. Consider the line $y = 3x + 1$, and the transformation T which is translation by the vector $(1, 1)$. What shape do you get when you apply T to each point on the line?

Reflections

26. Let T be reflection about the $x-$axis. Find the result of transforming the following points, and plot both the point and its reflection.

(A) $(-2, 3)$

(B) $(-5, -1)$

(C) $(1, -2)$

(D) $(3, 4)$

27. Let S be reflection about the vertical line $x = 3$. Find and plot $S(A)$, $S(B)$, and $S(C), S(D)$ on the same graph as before.

28. Let Q be reflection about the diagonal line $y = x$. Find and plot $Q(A)$, $Q(B)$, and $Q(C), Q(D)$ on the same graph as before.

29. Describe the set of all points $P = (x, y)$ such that $Q(P) = P$.

30. Find the length of the shortest path that starts at $(0, 5)$, touches the $x-$axis, and then ends at $(6, 3)$.

31. Find the length of the shortest path that starts at $(7, 1)$, touches the line $x = y$, and ends at $(13, 2)$.

32. Consider the line $y = 3x + 1$.

(i) What shape do you get when you apply T to each point on the line, where T i the reflection in exercise 26?

(ii) Repeat with S from exercise 27.

(iii) Repeat with Q from exercise 28.

Rotations

33. Let T be rotation counterclockwise by 90 degrees about the origin $(0,0)$. On a new sheet of grid paper, find the result of transforming the following points, and plot both the point and its reflection.

(A) $(-2, 3)$

(B) $(-5, -1)$

(C) $(1, -2)$

(D) $(3, 4)$

34. Let S be rotation counterclockwise by 90 degrees about the point $(1, -2)$. Find and plot $S(A), S(B), S(C), S(D)$.

35. Let R be rotation by 180 degrees about $(0, 2)$. Find and plot $R(A), R(B), R(C), R(D)$.

36. Let Q be rotation by 180 degrees about an unknown point $F = (x, y)$. If we're given $Q(2, 3) = (-4, 1)$, what is F?

37. Let P be rotation by 90 degrees clockwise about an unknown point $G = (x, y)$. If we're given $P(1, 7) = (-5, 8)$, what is P?

38. Consider the line $y = 3x + 1$.

(i) What shape do you get when you apply T to each point on the line, where T is the rotation in exercise 33?

(ii) Repeat with S from exercise 34.

6 Compositions

39. Let T be reflection about the $y-$axis, and S the reflection about the line $x = 3$. On a new sheet of grid paper, plot the points A, B, C, D. Then, plot $S(T(A)), S(T(B)), S(T(C)), S(T(D))$.

(A) $(-2, 3)$

(B) $(-5, -1)$

(C) $(0, -2)$

(D) $(3, 4)$

40. What is the symmetry $S \circ T$?

41. Let T be reflection about the $y-$axis, and R be reflection about the $x-$axis. Find and plot A, B, C, D, and $(T(R(A)), T(R(B)), T(R(C)), T(R(D))$.

1. $(5, 3)$

2. $(4, 0)$

3. $(0, 2)$

4. $(1, -3)$

42. What symmetry is $T \circ R$?

43. Let S be reflection about the line $x = 3$, and U be reflection about the line $y = 2$.

1. What are the **fixed points** of $U \circ S$?

2. What symmetry is $U \circ S$?

3. Does $S \circ U = U \circ S$?

44. Let T be reflection about the $y-$axis. What is $T \circ T$?

45. Let Q be reflection about the line $y = x$, and R be reflection about the $x-$axis. What symmetry is $R \circ Q$?

7   Formulas

Translation by the vector $(3, 5)$ can be written in the following form:

$$
\begin{aligned}
x &\rightarrow x + 3 \\
y &\rightarrow y + 5
\end{aligned}
$$

Similarly, reflection about the line $y = 2$ can be written:

$$
\begin{aligned}
x &\rightarrow x \\
y &\rightarrow 2 - y
\end{aligned}
$$

46. Write formulas for the following transformations:

(i) Translation by the vector $(4, -7)$

(ii) Reflection about the line $y = x$

(iii) Reflection about the line $x = -4$

(iv) Reflection about the line $y = -x + 1$

(v) Rotation 90 degrees counterclockwise about $(1, 3)$

(vi) Rotation 180 degrees about $(-2, 2)$.

47. Fill in the following table.

Transformation	Formula
Translation by the vector (a, b)	$x \;\rightarrow$ $y \;\rightarrow$
Reflection about the vertical line $x = a$	$x \;\rightarrow$ $y \;\rightarrow$
Reflection about the horizontal line $y = a$	$x \;\rightarrow$ $y \;\rightarrow$
Reflection about the diagonal line $y = x + a$	$x \;\rightarrow$ $y \;\rightarrow$
Reflection about the diagonal line $y = -x + a$	$x \;\rightarrow$ $y \;\rightarrow$
Rotation 90 degrees counterclockwise about (a, b)	$x \;\rightarrow$ $y \;\rightarrow$
Rotation 180 degrees about (a, b)	$x \;\rightarrow$ $y \;\rightarrow$
Rotation 270 degrees about (a, b)	$x \;\rightarrow$ $y \;\rightarrow$
The identity transformation	$x \;\rightarrow$ $y \;\rightarrow$

48. We define the following transformations:

- T is translation by $(2, -3)$

- R is reflection about the line $y = x - 1$

- H is reflection about the line $y = 3$

- S be rotation 90 degrees counterclockwise about $(-2, -1)$

- Q is rotation 90 degrees counterclockwise about $(3, 5)$

Use the table to compute and describe the result of the following compositions.

(i) $R \circ H$

(ii) $H \circ R$

(iii) $S \circ Q$

(iv) $S \circ T$

(v) $R \circ S$

Challenge Problems

(i) Find the length of the shortest path from $(2, 8)$ to $(6, 3)$ which touches the $x-$axis and the $y-$axis.

(ii) Find the length of the shortest path from $(1, 0)$ to $(2, 3)$ which touches the $y-$ axis and the line $x = 3$.

(iii) A triangle ABC is drawn such that the midpoint of A and B is $(0, 0)$, the midpoint of B and C is $(-1, 4)$, and the midpoint of C and A is $(-3, 1)$. Find A, B, and C.

(iv) A triangle ABC is drawn. Then, three squares on drawn outside the triangle, each square sharing one side with the triangle. The square with side AB has center $(0, 0)$, the square with side BC has center $(4, 6)$, and the square with side CA has center $(-6, 2)$. Find A, B, and C.

(v) Let R denote reflection about the line L given by the equation $y = mx + b$. If A is any point, and $R(A)$ its reflection, then L is the perpendicular bisector of A and $R(A)$. Use this fact to come up with a formula for R.

(vi) Let S denote rotation 60 degrees counterclockwise about the point $(0, 0)$. If A is any point, and $S(A)$ is its rotation, then the points $(0, 0), A, S(A)$ form an equilateral triangle. Use this fact to come up with a formula for S.